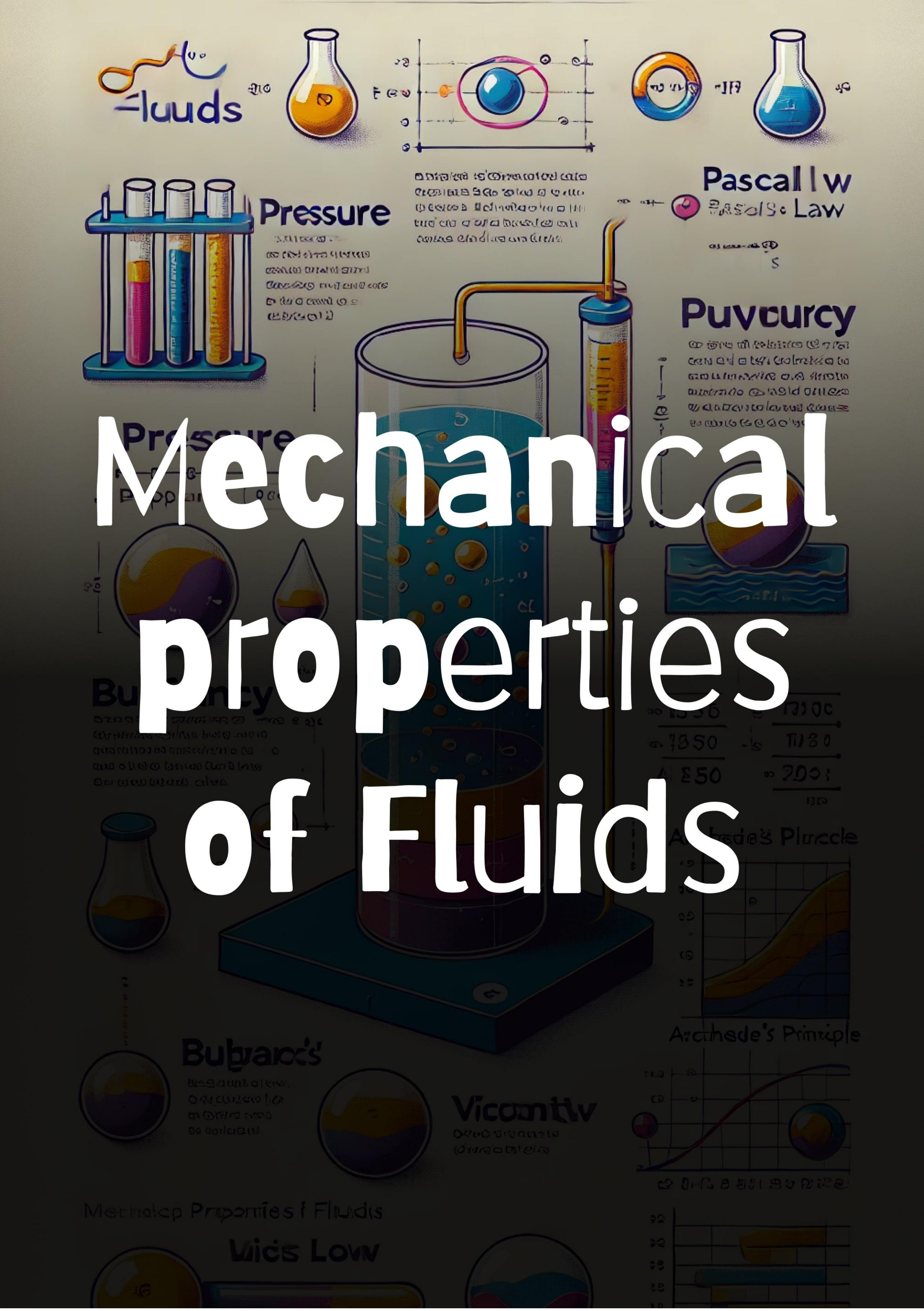
Mechanical properties of Fluids



1 cm = 10^{-2}

Mechanical Properties of fluids

(1) Mass - quantity of matter contained in a Body
- unit - 1 kg / gram
- Scalar

(2) Volume - 3-D Space Occupied by the matter
- unit - m^3
- litre - 1 litre = $10^{-3} m^3$
- Scalar

(3) Density -
- $\frac{mass}{Volume}$ unit = $\frac{kg}{m^3}$, $\frac{g}{cm^3}$
- Scalar

(a) linear mass density = $\frac{dm}{dl}$ kg/m

(b) Areal mass density = $\frac{dm}{dA}$ kg/m^2

(c) for liquid and gases
$\rho = \frac{dm}{dV}$ $\frac{kg}{m^3}$

- ★ $\rho_{water} = 1 \frac{gm}{cm^3} = 10^3 \frac{kg}{m^3}$

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Relative density (Specific gravity) $= \frac{\rho_{liquid}}{\rho_{water}}$

unit and dimensionless

* $\rho_{milk} = 1.04\ gm/cm^3 = 1.04\ kg/m^3$
* ρ_{Hg} (Heaviest liquid) $13.6\ gm/cm^3 = 13.6 \times 10^3\ kg/m^3$

* $\rho_{Ice} = 0.9\ gm/cm^3$ * $\rho_{oil} = 0.8\ gm/cm^3$

$\rho_{oil} < \rho_{water}$

Density of mixture of two liquid:

$\rho_{mix} = \frac{\text{total mass}}{\text{total Volume}} \quad \frac{m_1 + m_2}{V_1 + V_2}$

* Towo liquid Having mass m_1 and m_2 and density ρ_1 and ρ_2 are mixed then find density of mixture

$d_{mix} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$

* mass same of different liquid

$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

* Volume same of two different liquid

$$\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$$

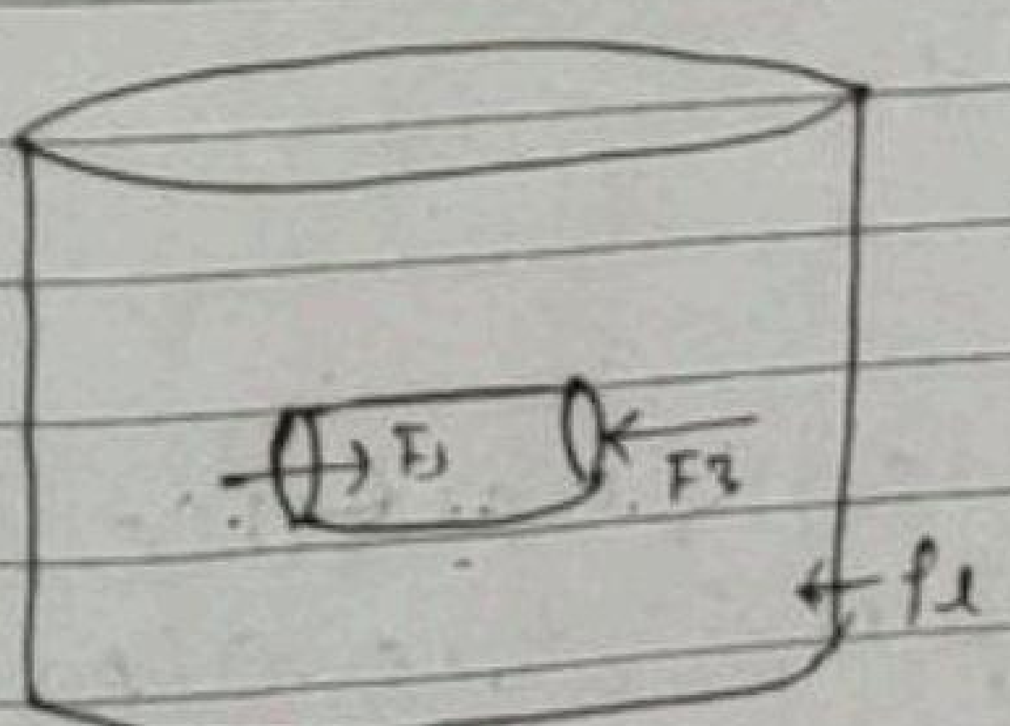
$$\rho_1 < \rho_{mix} < \rho_2$$

Pressure

$$P = \frac{Force}{Area}$$

Scalar
unit 1 Pascal
air pressure atmospheric Pressure
Pearth surface = (1 atm = 10^5 Pascal)

→ as we move upward from ground atm. pressure decreases



F.B.D of Imaginary cylinder

$F_1 - F_2 = ma$
$F_1 - F_2 = 0$
$F_1 = F_2$
$P_1 A = P_2 A$
$P_1 = P_2$

→ liq. pressure is same at all Horizontal position.

$mg - \rho V a$ $\quad \rho A l g$ $\quad mg$ $\quad \rho V g$ $\quad PVg$ $\quad \rho A$

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FBD in y-axis of Imaginary cylinder of liquid.

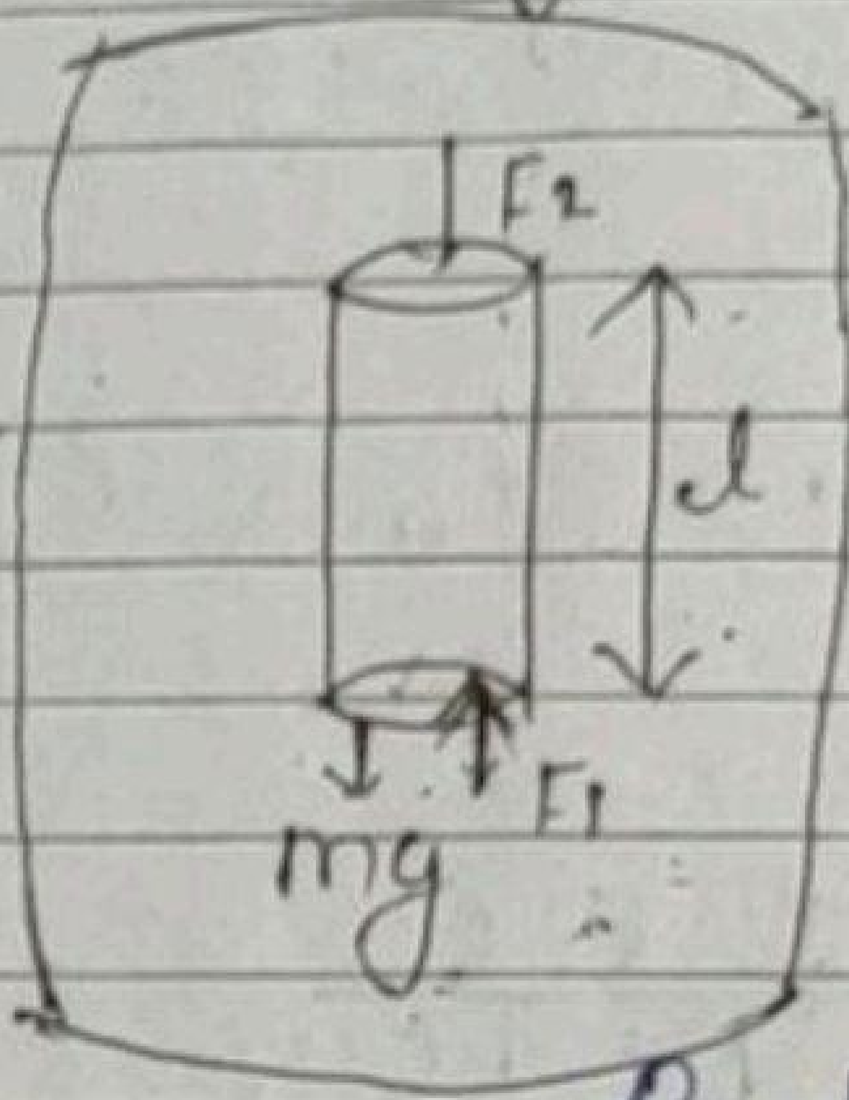
$F_{net}\ y = 0$

$mg + F_2 = F_1$

$\rho A l g = F_1 - F_2$

$\rho A l g = P_1 A - P_2 A$

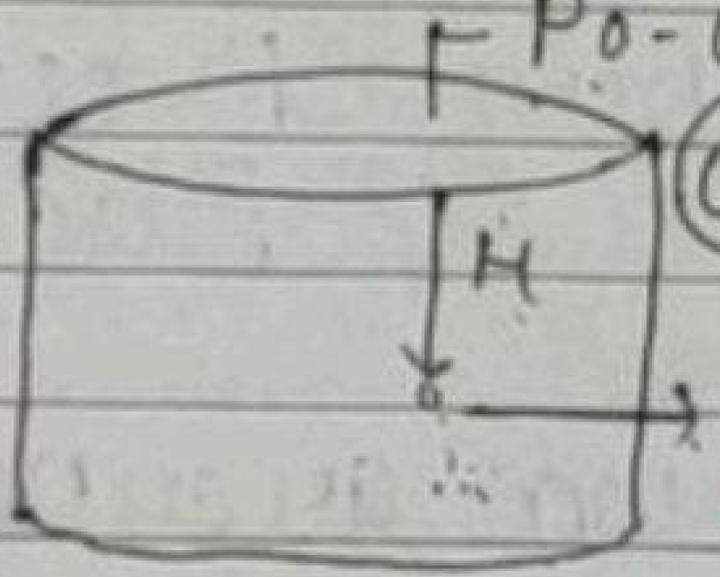
$P_1 - P_2 = \rho g l$



If a is given

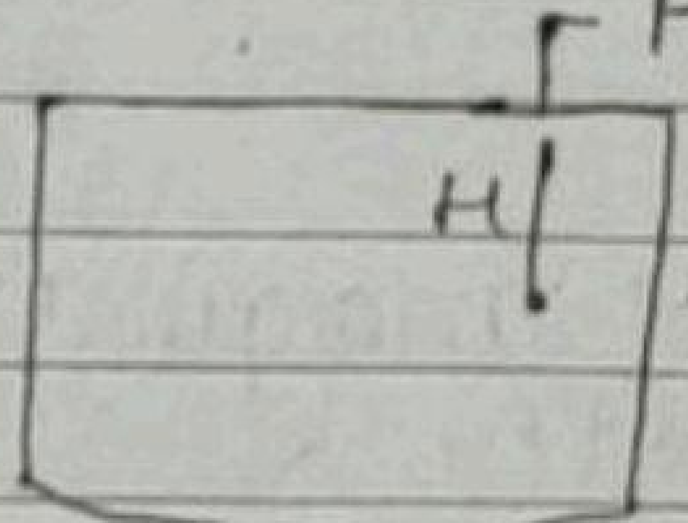
$P_1 - P_2 = \rho l (g + a)$

upward force ↓ Hoga
downward ↑ Hoga
same Height same pressure



P_0 - atm P (Open)

$P_{net} = P_0 + \rho g H$



P_0 - atm P (closed)

$P_{net} = \rho g H$.

Q) Find Pressure exerted pressure below a column of water, open to the atmosphere

A for 10 m Height $\quad P_{net} = P_{atm} + \rho g h$
$\rho g h = 1\ atm$
$= 1\ atm + 1\ atm$
$= 2\ atm$

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~~10 = 1 atm~~

20m $\quad$ 20 = 2 atm

$P_{net} = P_{atm} + \rho g h$

$1 atm + 2 atm = 3 atm$

Q) The pressure at the bottom of a water tank is 4P where P is atm pressure. If water is drawn out till water level decreases by 3/5 th, then pressure at the bottom of the tank.

$P + \rho g H = 4p$

$\rho g H = 3p$

$H' = H - \frac{3}{5}H = \frac{2H}{5}$

$P_{Bottom} = P_0 + \rho g \left(\frac{2H}{5}\right)$

$= P_0 + 3p\left(\frac{2}{5}\right) = \frac{5p + 6p}{5}$

$P_{Bottom} = \frac{11}{5}p$

Q) If pressure at the half depth of a lake is equal to 3/4 times the pressure at its bottom, then find the depth of the lake.

$P_{Half\ depth} = \frac{3}{4}(p_{bottom})$

$P_0 + \frac{\rho g H}{2} = \frac{3}{4}(P_0 + \rho g H)$

$4P_0 + 2\rho g H = 3P_0 + 3\rho g h$

$4P_0 - 3P_0 = 3\rho g H - 2\rho g H$

$P_0 = \rho g H$

$10^5 = 10^3 \times 10 \times H$

$H = 10m$

$\frac{4}{3}\pi r^3$ $\quad V \propto r^3$ $\quad \frac{4}{3}\pi r^3$ $\quad V \propto r$

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Q) A air bubble rises from bottom of a lake to surface. Its radius Increases by 200% and atmospheric pressure is equal to water column of Height H, then depth of lake is :

from same

r Initial $= r$.

r final $= 300\% \, r$

$r_f = 3r$.

$V \propto r^3 \; 27$

$V = 27V$

$PV = \text{const}^n$

(PV) Initial $= (PV)$ final

$(P_0 + \rho g h)V = P_0 27V$

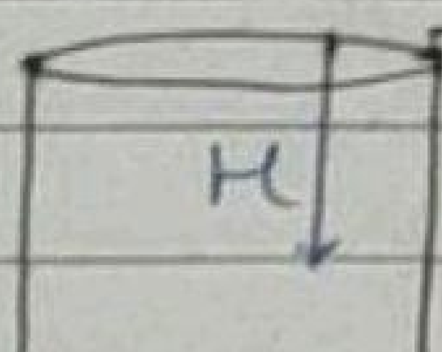
$P_0 + \rho g h = 27 P_0$

$\rho g h = 26 P_0$

$\rho g h = 26 \rho g H$

$h = 26H$

Q) A tank is filled by liquid of density ρ upto Height H. The avg pressure on the walls of container is.



$P_{atm} = 0$

$P_{avg} = \frac{P_i + P_f}{2} = \frac{0 + \rho g h}{2} = \frac{1}{2} \rho g H$

Ø. pressure doesn't depend upon amount of liquid and shape of container.

Q) The Volume of an air bubble is doubled as it rises from the bottom of lake to its surface. The atmospheric pressure is 75 cm of mercury. The ratio of density of mercury to that lake water is 40/3

The depth of the the lake, in Nature is:

$\frac{\rho_H}{\rho_W} = \frac{40}{3}$ $\quad (P_0 + \rho g H)V = P_0 2V$

$P_0 = \rho_{Hg}\, g\, 75\,cm$ $\quad \rho_W g H = P_0 = \rho_{Hg}\, g\, 75$

$H = \frac{\rho_{Hg} 75}{\rho_W} = \frac{40 \times 75}{3}$

$= 1000\,cm$

$10\,m$

Pascal law:

→ Pressure in a fluid at rest is same at all points which are at same Height.

→ An extra pressure applied at any point of enclosed fluid is transmitted undiminised to every point of the fluid and the walls of containing vessel.

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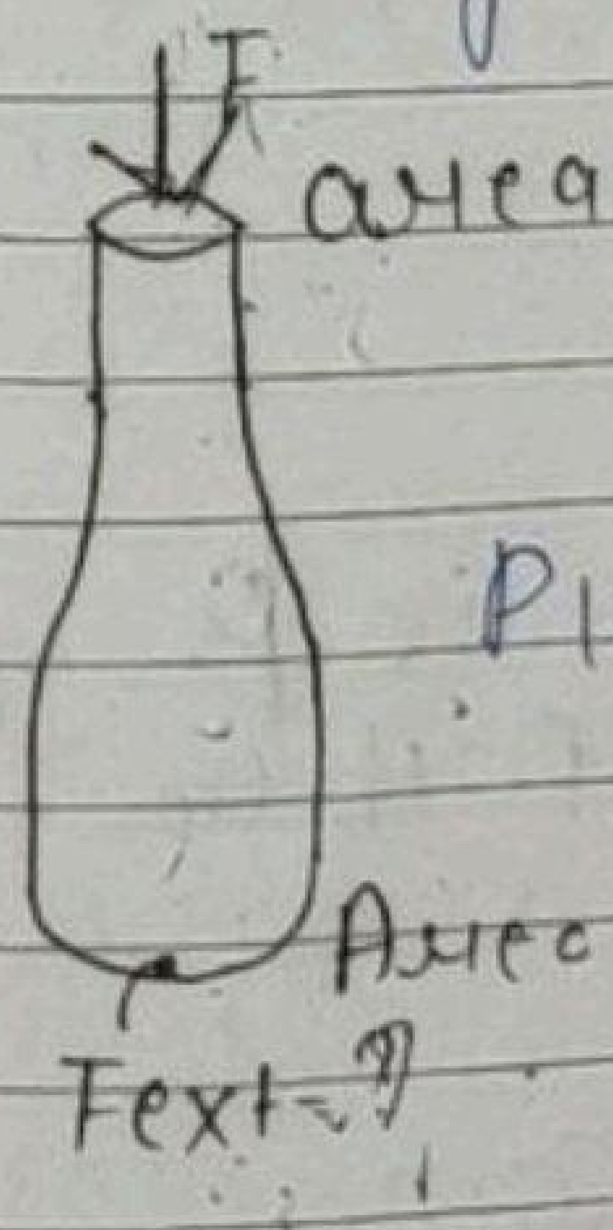
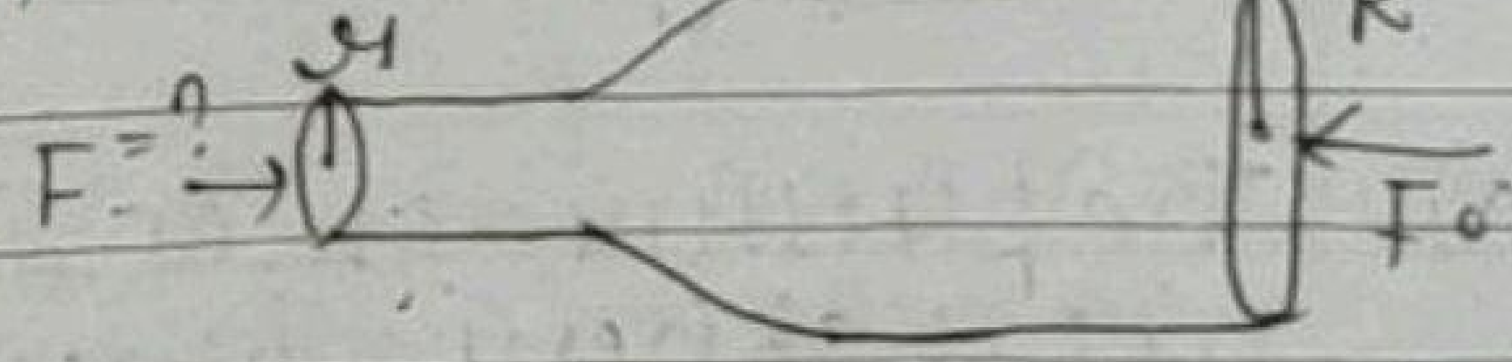


$\frac{F}{a} = \frac{F'}{A}$

$P_1 = P_2$

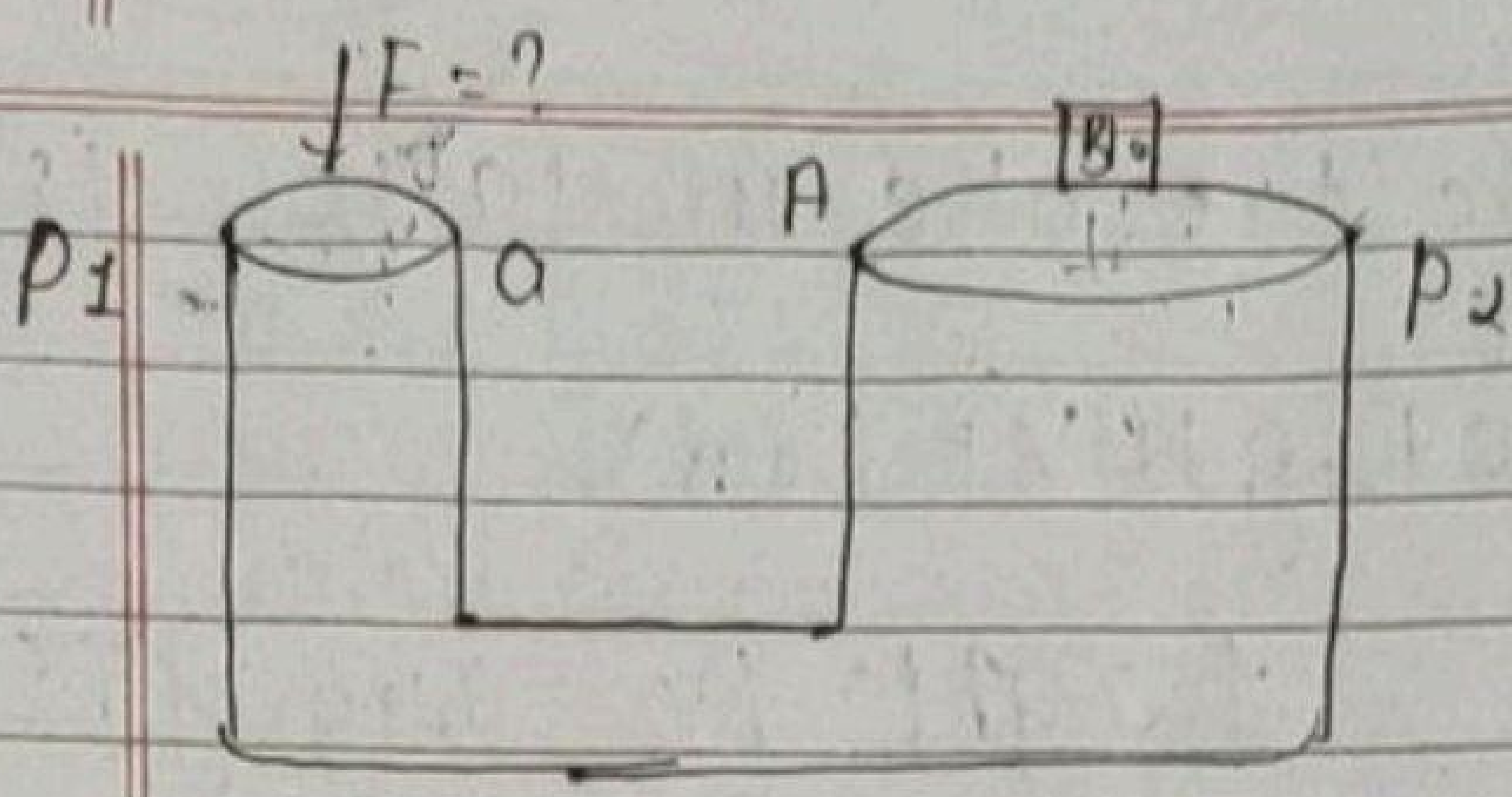
#



$P_1 = P_2$

$\frac{F}{\pi r^2} = \frac{F_0}{\pi R^2}$

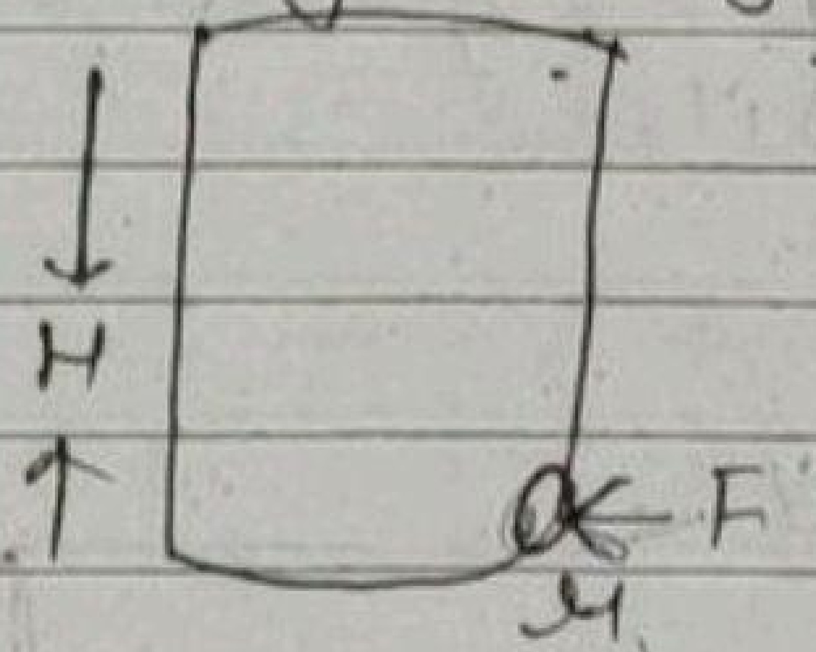
$F = \frac{r^2 F_0}{R^2}$



$P_1 = P_2$

$\frac{f}{a} = \frac{mg}{A}$ weight, $mg = \frac{FA}{a}$

Q) A large vessel of Height H, is filled with a liquid of density ρ, upto the brim. A small Hole of radius r is made at the side vertical face, close to the base. The Horizontal force is required to stop the gushing of liquid is



F = P x area

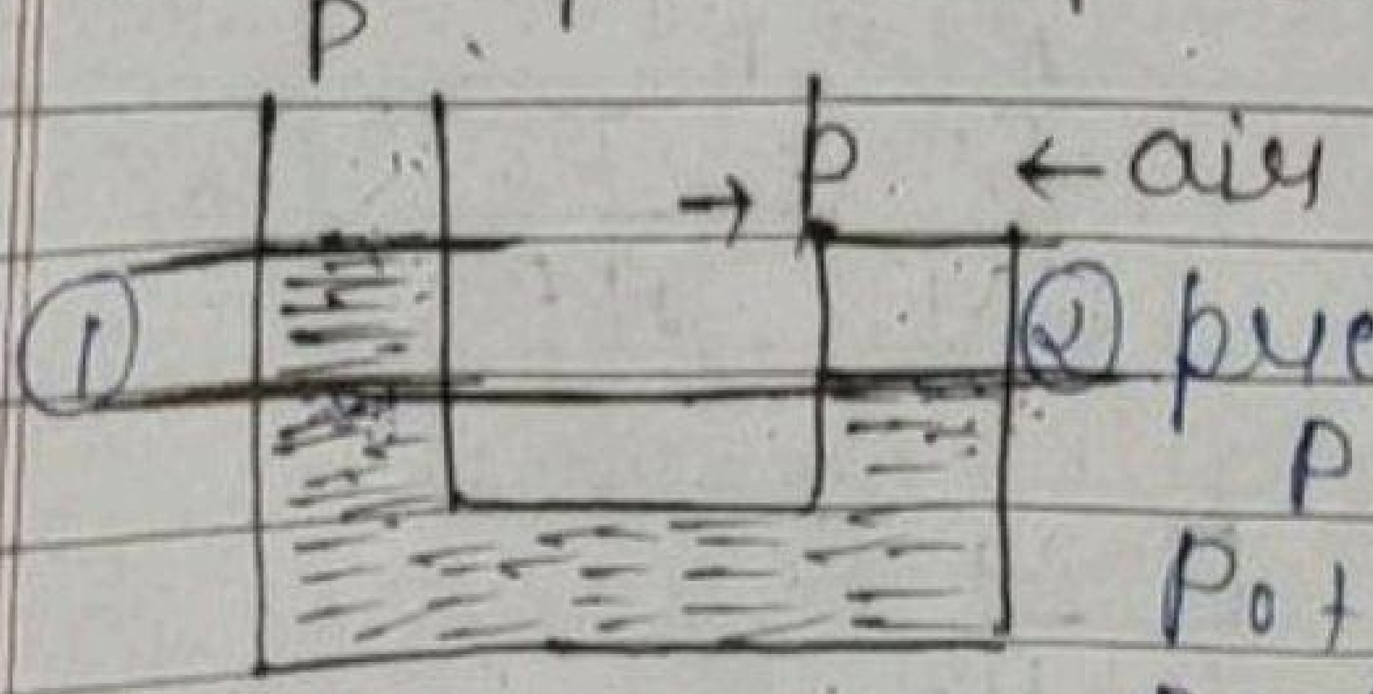
$\rho g H \times \pi r^2$

$P_1 = P_2$

$\frac{F}{\pi r^2}$

$F = \rho g h r^2$

Q) The pressure of confined air is p. if the atmospheric pressure is P then



pressure same

$P_1 = P_2$

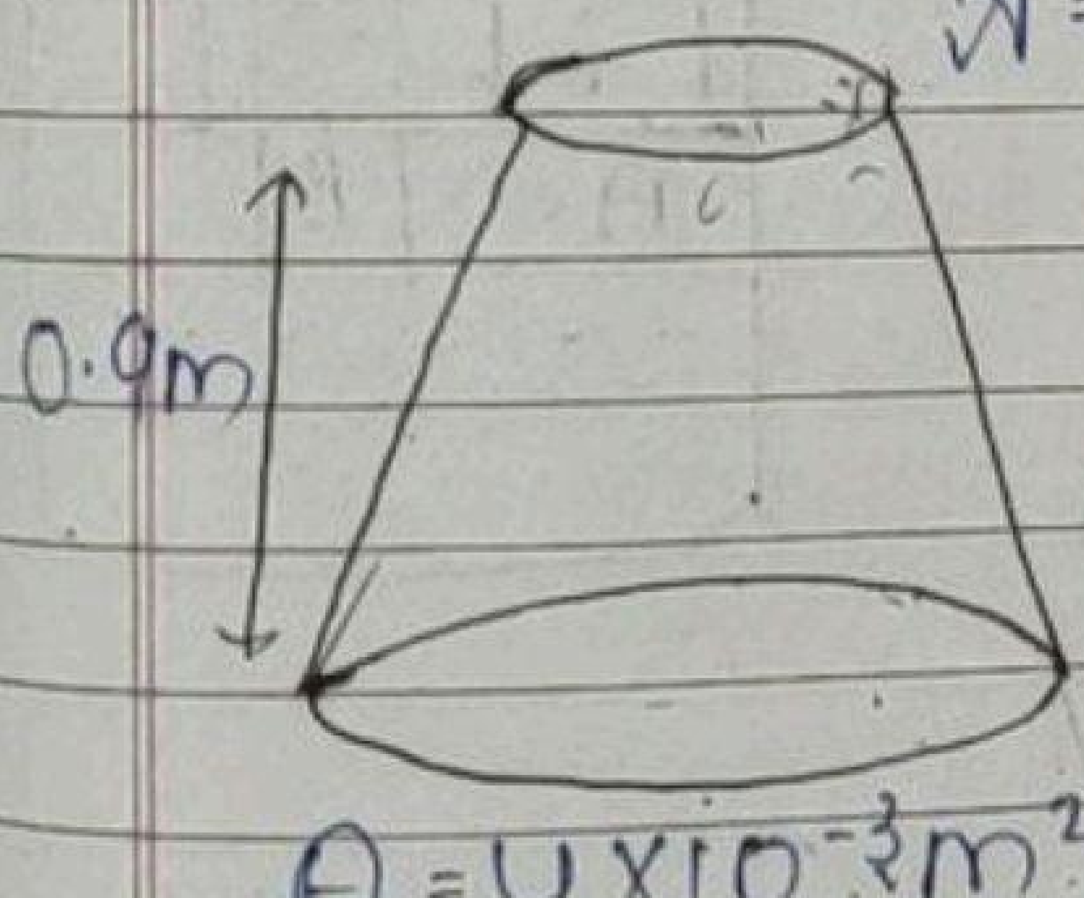
$P_0 + \rho g h = p$

$P + \rho g h = p$

b) P

Pnet = Pliquid + P_0
↓
gaugepressure - can be
⊙ Positive
⊙ Negative
⊙ Zero

Q) A uniform tapering vessel is filled with a liquid of density of 1000 kg/m³ what is the force acting on the basis of the vessel due to the liquid?

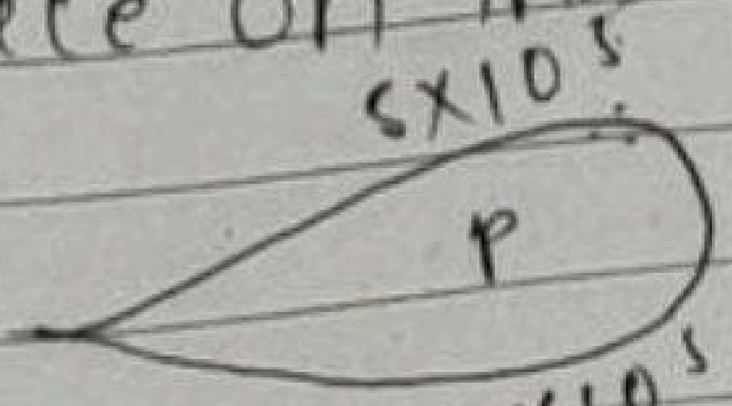


$A = 2\times10^{-3} m^3$

$P = \rho g h = 10^3 \times 10 \times \frac{9}{10}$

$F = P \times A$
$= 10^3 \times 9 \times 4 \times 10^{-3}$
$= 36 N$

Q) In a wind tunnel experiment, the pressure on the upper and lower surface of the wings are 5×10^5 Pa and 10×10^5 Pa. If the area of wing is 80 m², the net lifting force on the wing is



$F = \Delta P \times A$
$5\times10^5 \times 80$
400×10^5

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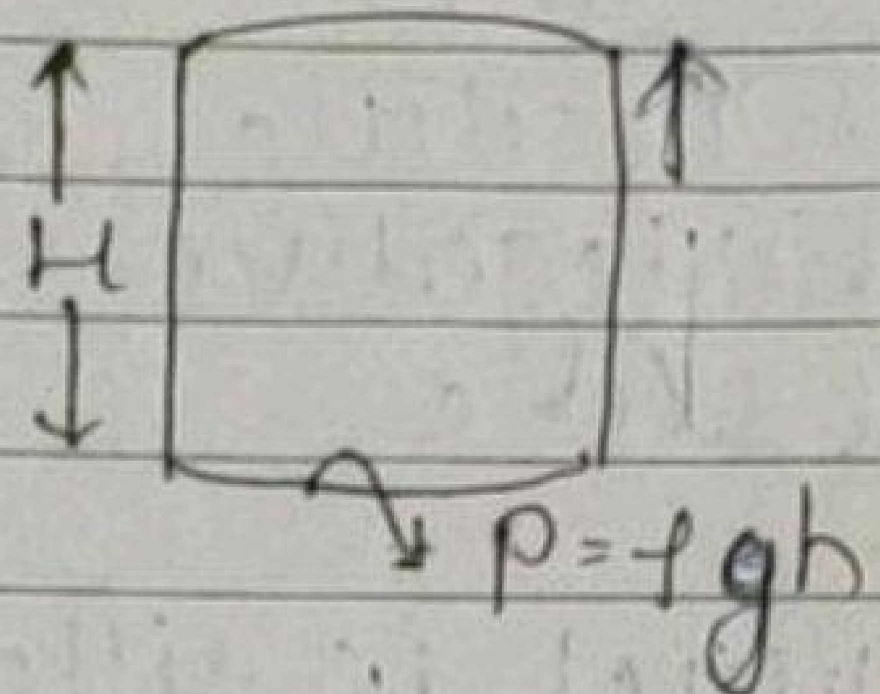
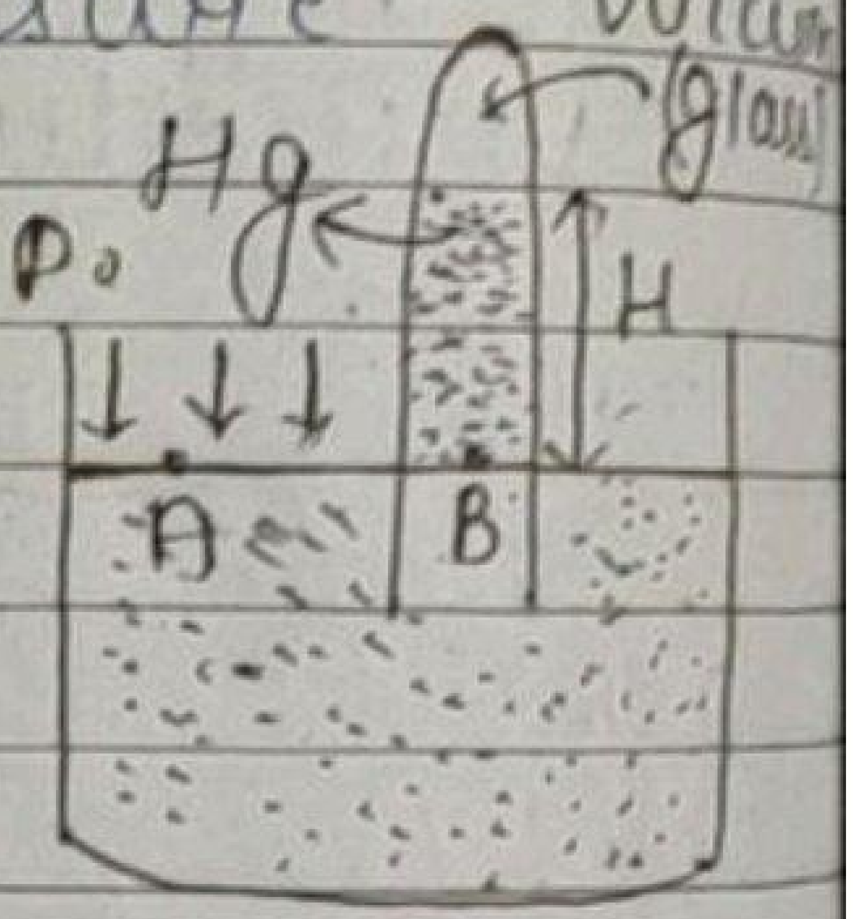
Q) A Beaker containing a liquid of density ρ moves up with an acc^n 'a' The pressure due to the liquid at a depth h



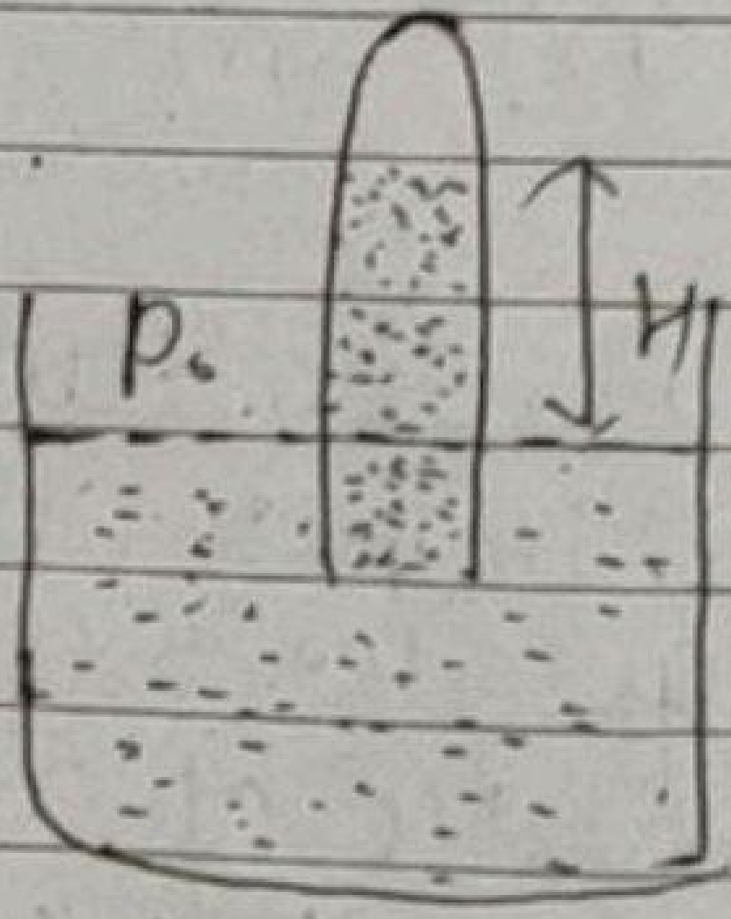
$P = \rho g h$
$= \rho (g+a) h$

Barometer: Instrument use to measure atmospheric pressure



$P_A = P_B$
$P_0 = \rho g h$
$P_0 = 13.6 \times 10^3 \times 10 \times 76 cm$
$1.01 \times 10^5 N/m^2$
$P_0 = 1.01 \times 10^5 Pa = 1 atm$

Ramlal gareeb aadmi
water Barometer



$P_0 = \rho g h$

$1.01 \times 10^5 = 10^3 \times 10 \times h$

$H = 10.1 m$

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Q) The atmospheric pressure at a place is 10^5 Pa. If tribromomethane (R.D = 2.9) be employed as the barometric liquid, the barometric Height is

$P_0 = \rho g h$

$10^5 = 2.9 \times 10^3 \times 10 \times h$

$H = \frac{10}{2.9}$

Q) A barometer is constructed using a liquid density = 760 kg/m³. What would be the Height of liquid column, when a mercury barometer reads 76 cm (ρ = 13600 kg/m³)

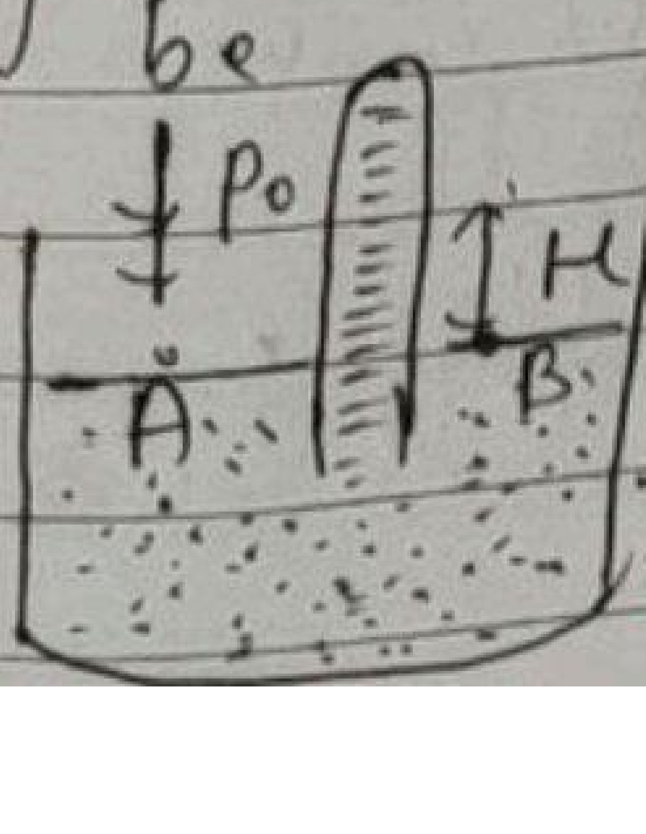
$P_0 = \rho g h$

$10^5 = 760 \times 10 \times h$

$\frac{1000}{76} = h$

$\frac{100}{7.6} = h$

Q) A barometer kept in an elevator reads 76 cm when it is at rest. If the elevator goes up with one accⁿ, the reading will be



rest
$P_A = P_B$
$[P_0 = \rho g h]$

accⁿ upward
$P_A = P_B$
$P_0 = \rho(g+a)h$

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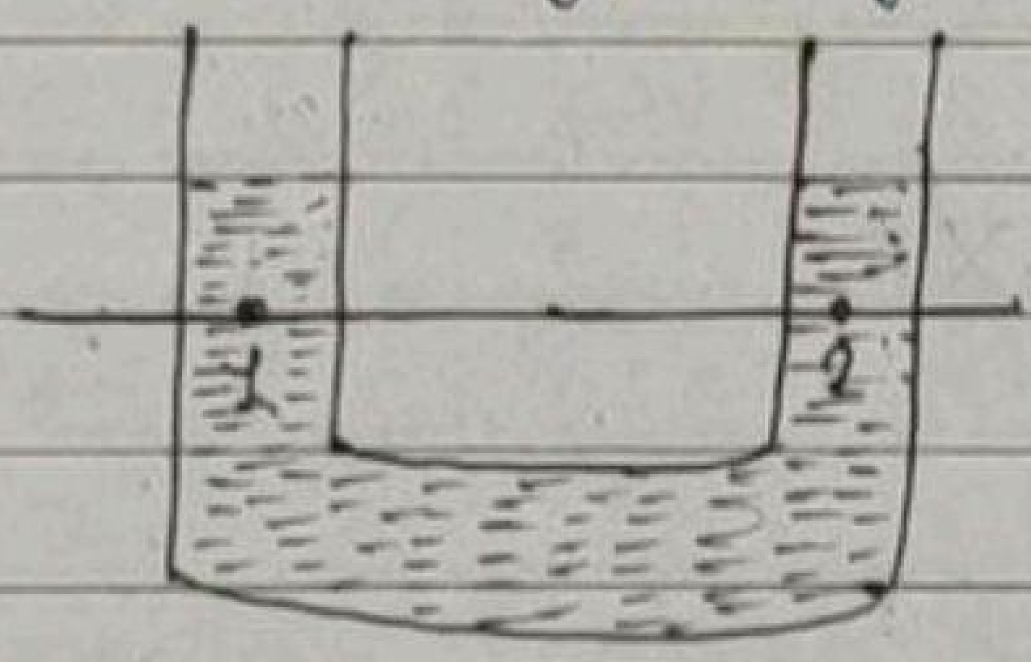
Q). A barometer kept in an elevator reads 76 cm when the elevator accn upward. The most like pressure Inside the elevator?

$1 \cdot P_0 = \rho a$
$P = \rho(g+a)h$

$P_0 = \rho g h$
$\rho g h = \rho(g+a) h$
$\rho g h = \rho(g+a) 76$

$\rho g h = (g+a) 76 = 77$

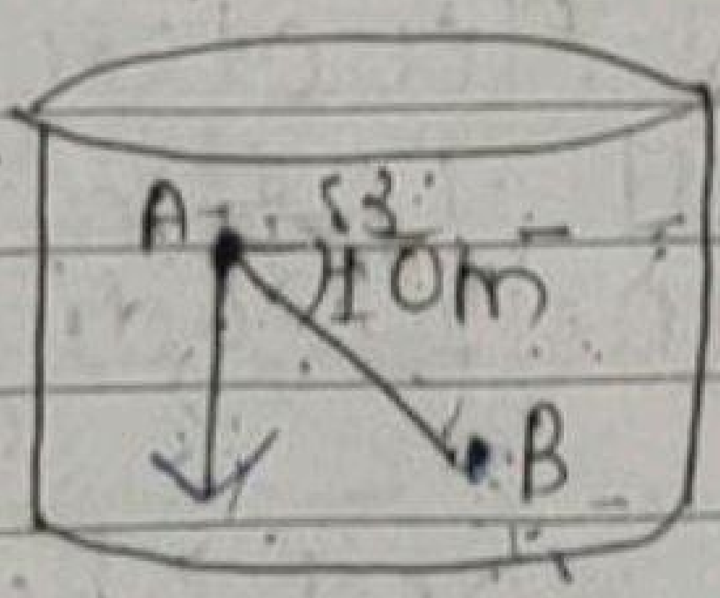
Questions on U-Tube-
Pressure of same liquid at same Height of same point is same



★ candel wala
★ coin wala

$P_1 = P_2$

Q)

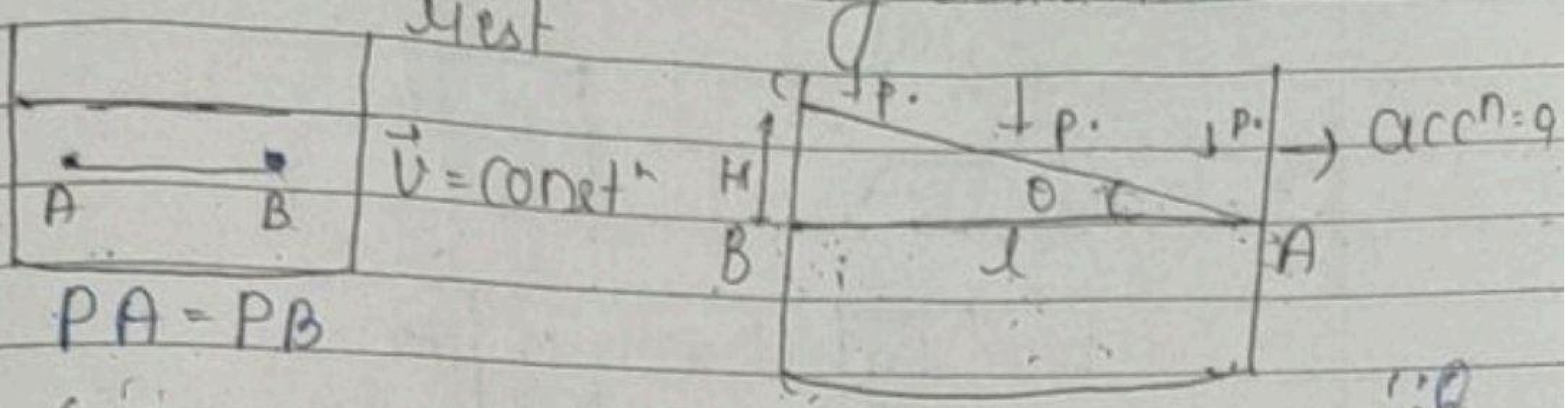


$P_B - P_A = \rho g h_{vertical}$
$= 10^3 \times 10 \times 10 \sin 53°$
$10^4 \times \frac{4}{5} \times 10$

$= 10^5 \times \frac{4}{5}$

Horizontal accelerating container.

rest



$P_A = P_B$

$\rho a l = \rho g h$

$\left[\tan\theta = \frac{a}{g} = \frac{l}{h}\right]$ (II)

$P_B - P_A = \rho a l$

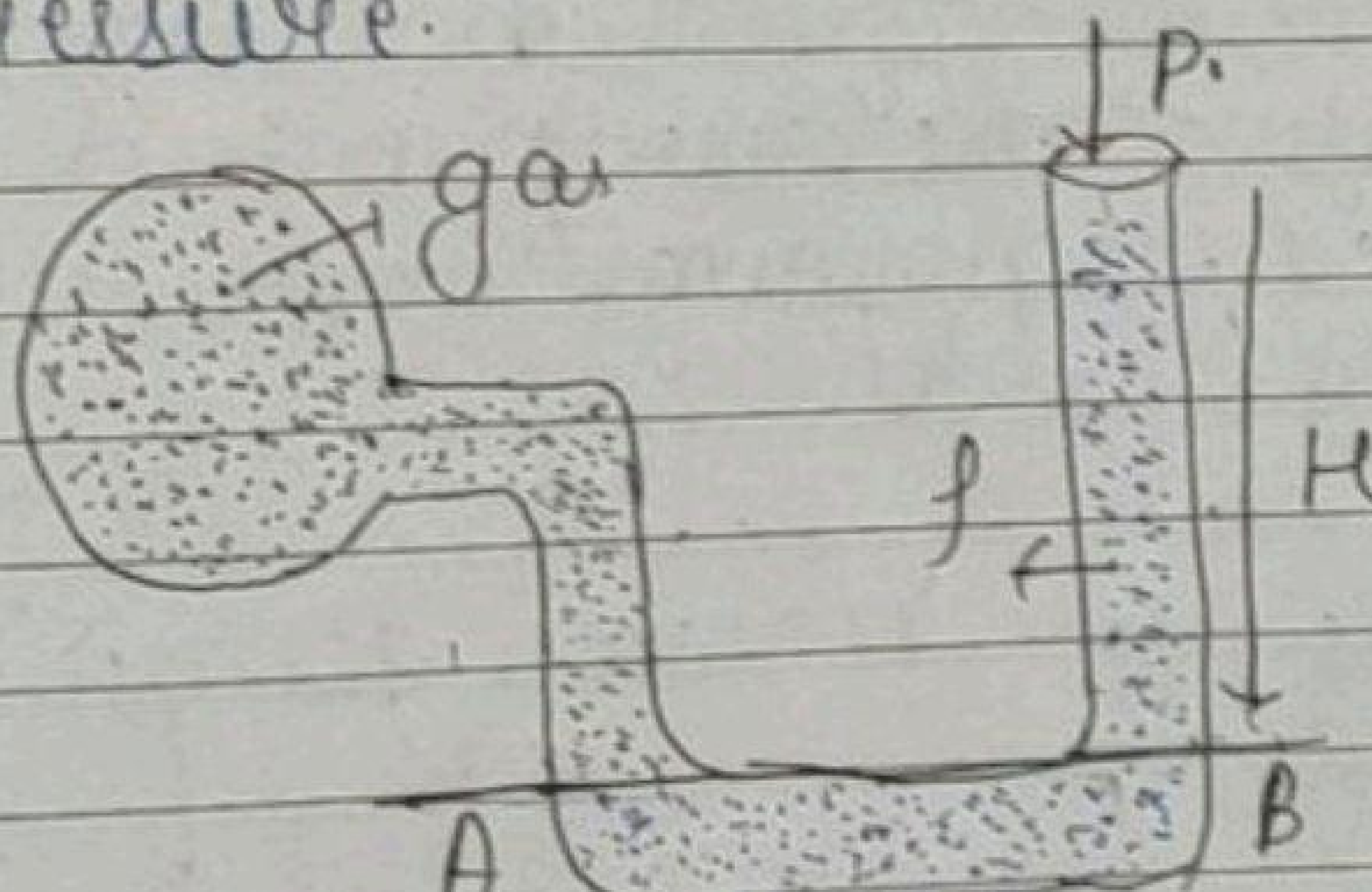
$P_B - P_C = \rho g H$

$[P_A = P_C = P_0]$ (I)

Manometer:

→ Is an Instrument used to measure gas pressure.

Case-(I)



$P_A = P_B$

$P_{gas} = P_0 + \rho g h$

★★★ a↑ $P_{difference} = \rho h(g+a)$

a↓ $P_{difference} = \rho h(g-a)$

★ upar jaenge to Heigh kam hoga $\left[H \propto \frac{1}{g_{eff}}\right]$

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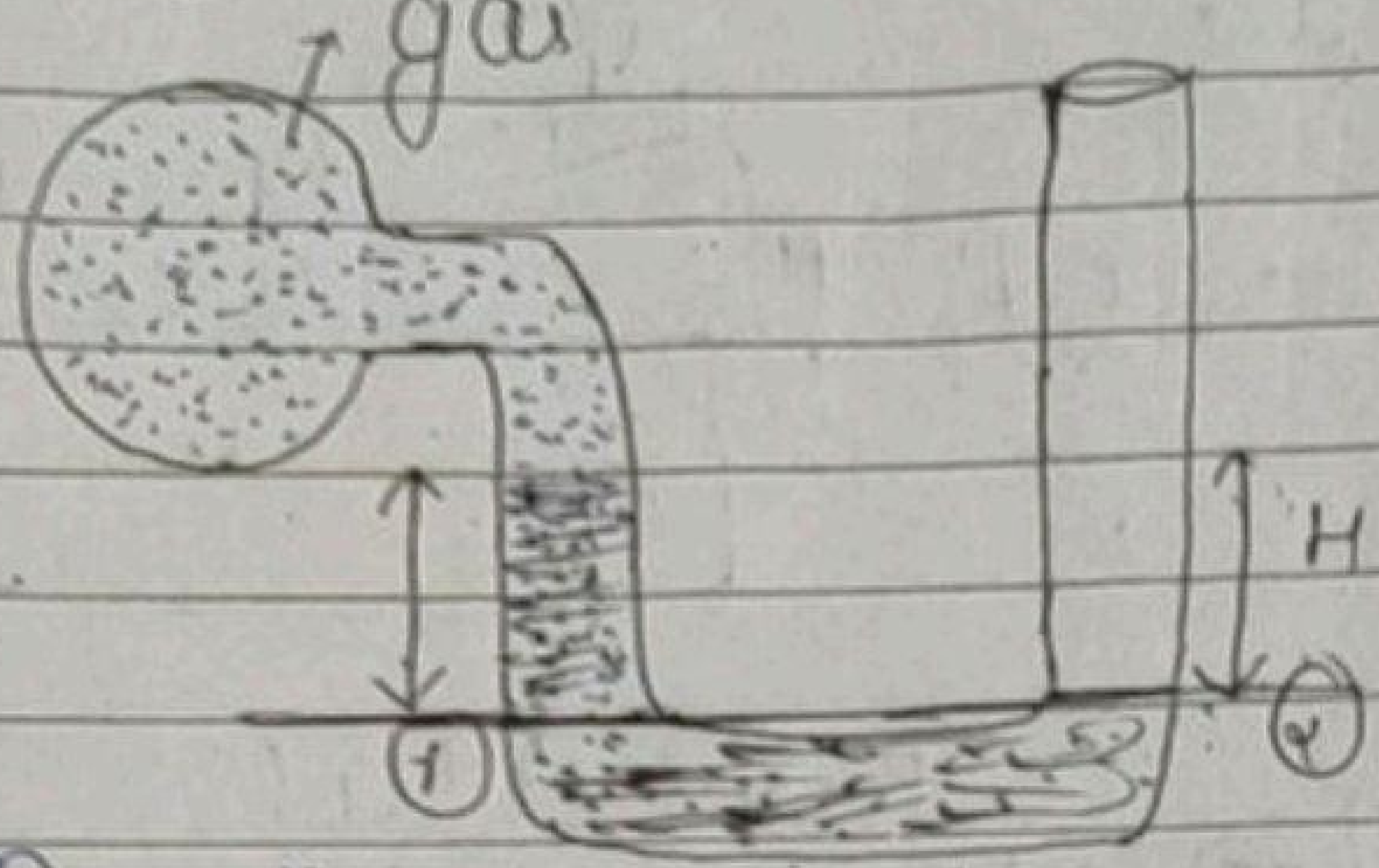
Case (II)



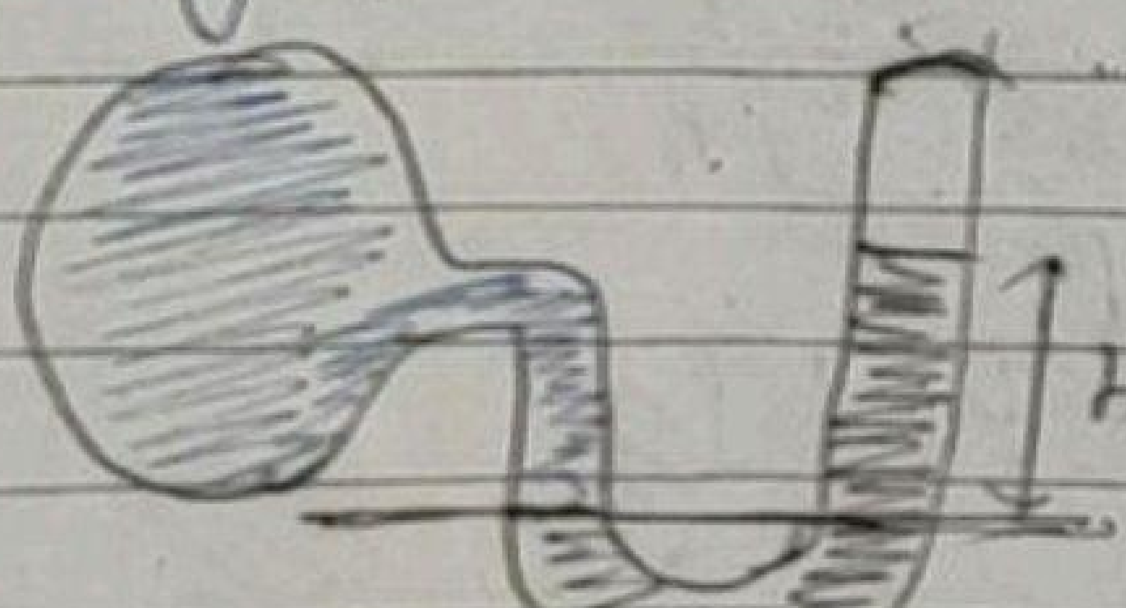
$P_1 = P_2$

$P_{gas} + \rho g H = P_0$

$P_{gas} = P_0 - \rho g H$

Q) A manometer tube contains mercury of density $13.6 \times 10^3\ kg\ m^{-3}$ what difference in the level of mercury in two arms is indicated by gauge pressure of 1.03×10^5 Pa

gauge pressure = closed = P liquid



$P_{gas} = P_0 + \rho g h$

$P_{gas} = \rho g h$

$\rho g h = 1.03 \times 10^5$

$13.6 \times 10 \times 10^3 \times H = 1.03 \times 10^5$

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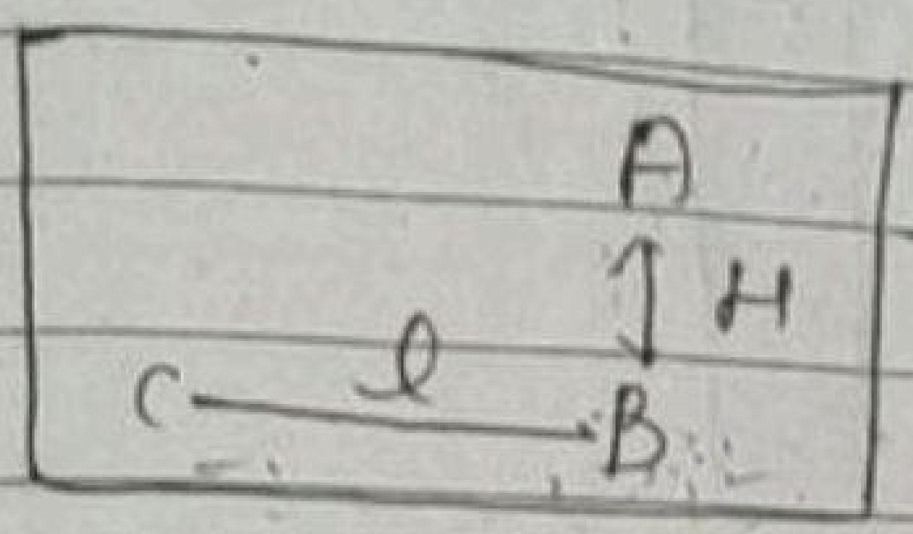
$H = 75.5\,cm$



pressure difference b/w C and A

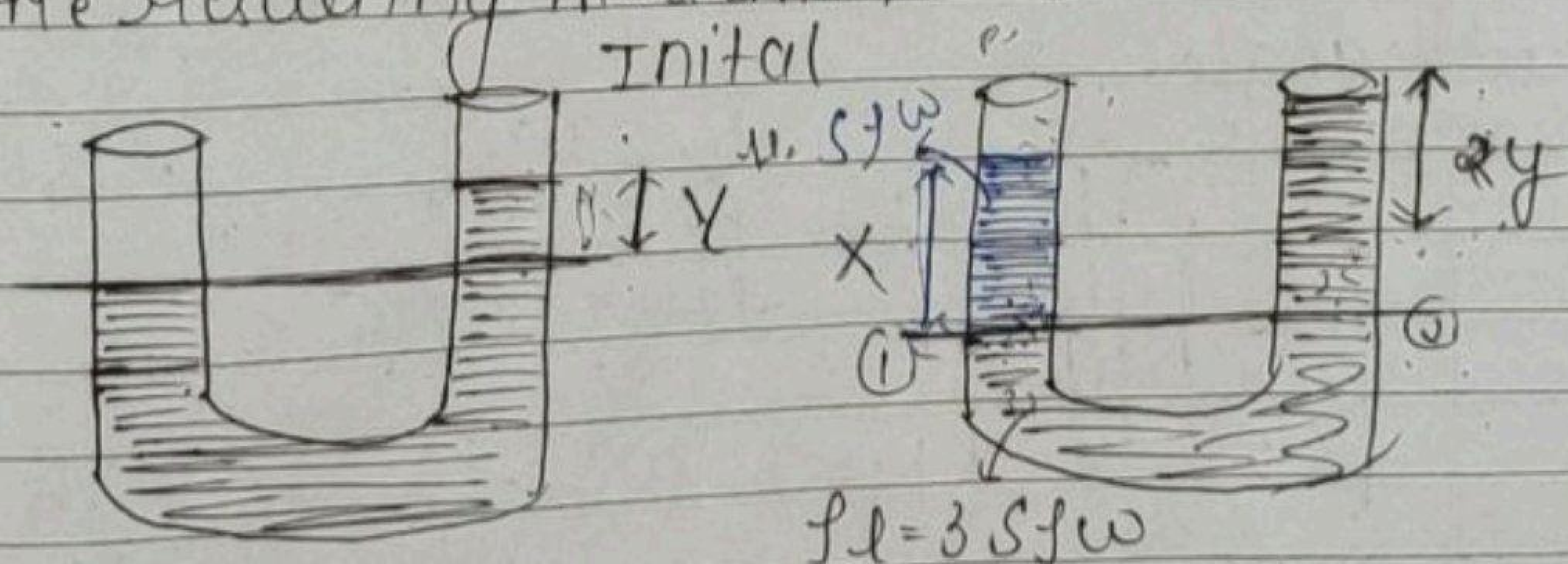
$P_B - P_A = \rho g H$

$P_C - P_B = \rho a l$

$P_C - P_A = \rho g H + \rho a l$

$P_C - P_A = \rho [gH + al]$

Q) Initially filled the liquid of specific gravity 3S. A lighter liquid of specific gravity S is poured into one of limb such that the length of column of lighter liquid is x. What is the resulting movement?



$\rho_l = 3S\rho_w$

$P_1 = P_2$

$P_0 + S\rho_w g x = P_0 + 3S\rho_w g 2y$

$x = 6y \quad y = x/6$

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① **Archimedes Principle**: Pressure difference in a fluid

② F_B Buoyant force / upthrust = weight of displaced liq. = $\rho V_{im} g$ ($\rho_e V_s g$)

③ Volume of liq. displaced by solid = Volume of solid inside liquid

④ appt weight (N) = $Mg - F_B$
F_B = wt of displaced liq.

$F_{net} = \rho V g$

→ doesn't depend on area of object and Height of object

densi ~~Volume~~ of solid Inside liquid ⊘

densi ~~Volume~~ of displaced liquid (ρ)

Statement:
The loss of weight of a body submerged (partially or fully) in a fluid is equal to the weight of fluid displaced.

(T.) Case-(I)

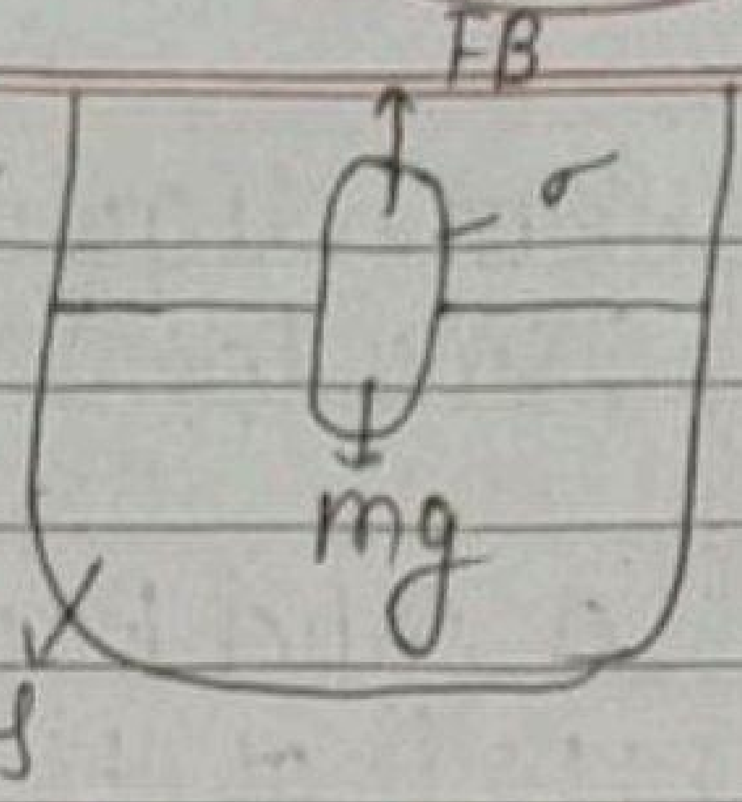
$\sigma_{object} = \rho_{liquid}$ apparent wt zero

$mg = F_B = \rho V g$

app. wt = 0

$\rho V_{total}\, g = \rho V_{In} g$

$V_{total\ object} = V_{Inside}$



body will float completely submerged

(2.) Case-②

$\sigma_{object} < \rho_{liquid}$ ✱ apparent wt is zero

$mg = F_B$

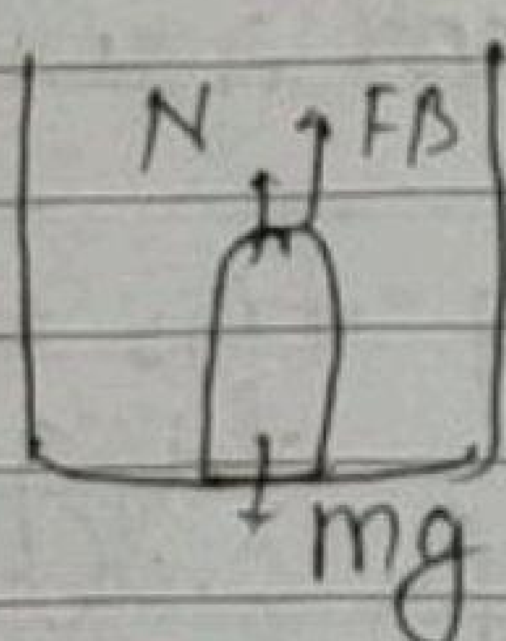
$\sigma V_{body} g = \rho V_{Inside} g$

$100 \times \left[\frac{V_{Inside}}{V_{object}}\right] = \left[\frac{\sigma}{\rho}\right] \times 100$

floating with partially submerged

(3) Case (III)

$\sigma_{object} > \rho_{liquid}$



Completely sink

$N = mg - F_B$

$\sigma V g - \rho V g$

$= \sigma V g \left[1 - \frac{\rho}{\sigma}\right]$

$N = mg\left[1 - \frac{\rho}{\sigma}\right]$ when sink

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★ Same liquid same object with different orientation have same byont force

Q) A Solid Body floating in water Has 1/5th of its Volume Immersed in it what fraction of its Volume will be Immersed, if it floats in a liquid of specific gravity 1.2.

(1) water

$$\frac{V_{Inside}}{V_{outside}} = \frac{\sigma}{\rho_w}$$

$$\frac{1}{5} = \frac{\sigma}{\rho_w}$$

$$\sigma = \frac{\rho_w}{5}$$

(2) liquid

$$\frac{V_{In}}{V_{ow}} = \frac{\sigma}{\rho_l} = \frac{\rho_w}{5 \times 1.2}$$

$$\frac{V_{In}}{V_{ow}} = \frac{1}{6}$$

Q) A cube of edge length 10 cm is just balanced at the interface of two liqu. A and B. If A and B has R.D 0.6 and 0.4 then mass of cube

$$mg = F_{B1} + F_{B2}$$

$$= \rho V g + \rho V g$$

$$mg = 0.6 \rho_w A \underset{cm}{4} g + 0.4 \rho_w A \underset{cm}{6} g$$

$$= \frac{6}{10} \times 400 + \frac{4}{10} \times 600$$

$$m = 240 + 240 = 400$$

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A solid of density 2.5 kg m^{-3} floats in a fluid with one-third of its volume immersed in it what is the density of fluid.

$\sigma = 2.5$

$\frac{V_{In}}{V_{out}} = \frac{\sigma}{\rho}$

$\frac{1}{3} = \frac{2.5}{\rho}$ $\quad \rho = 3 \times 2.5 = 7.5$

4

Q. The density of Ice is 0.9 g/cm^3. what % by volume of the block of ice floats outside the water?

$\left[\frac{V_{In}}{V_{out}} \times 100\right] = 0.9 \times 100 = 90\%$

90% submerged in water

Outside the water ⇒ 10%

Q) A block of steel of size $5 \times 5 \times 5$ cm^3 is weighed in water. if rel. D of steel is 7, find its apparent weight.

$N = mg\left(1 - \frac{\rho_w}{\sigma_{ob}}\right)$

$7\rho_w \times 5 \times 5 \times 5 \times g\left[1 - \frac{\rho_w}{7\rho_w}\right]$

$5 \times 5 \times 5 \times 7\rho_w g\left[\frac{7-1}{7}\right]$

$5 \times 5 \times 5 \times 6\rho_w g$

Q) A block of wood floats in water with $\frac{4}{5}$th its volume submerged, but it just floats in another liquid. The density of liq in kg/m³

first case

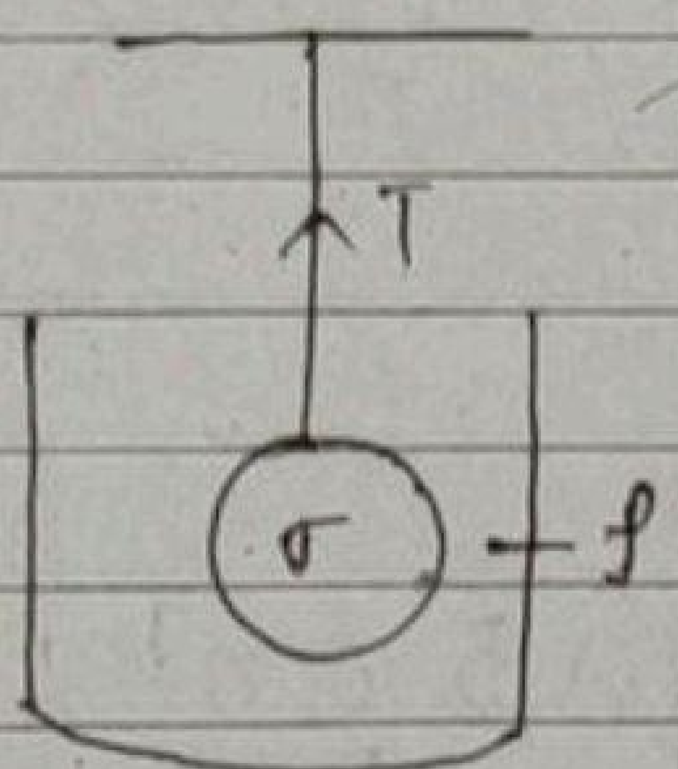
$$\frac{V_{in}}{V_{out}} = \frac{\sigma}{\rho_w}$$

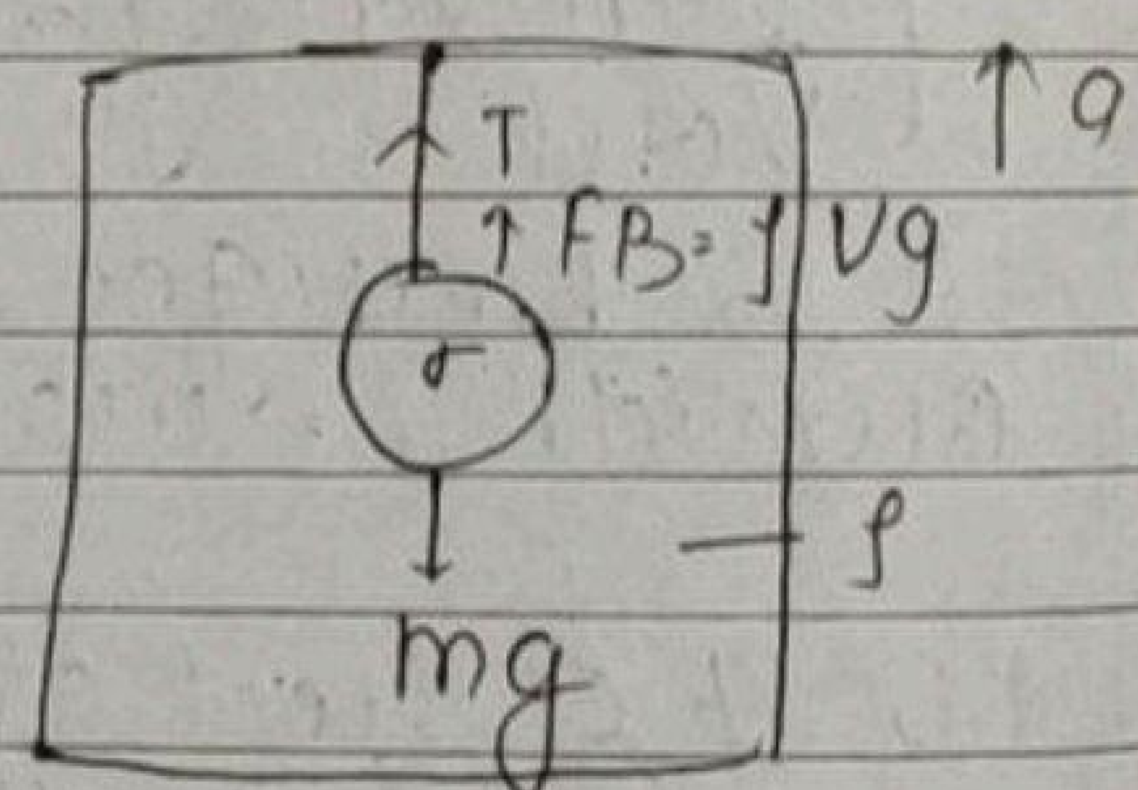
$$\frac{4}{5} = \frac{\sigma}{\rho_w} \qquad \sigma = \frac{4}{5}\rho_w$$

Dusre mein float: $\sigma = \rho_{liq}$

$$\rho_{liq} = \sigma = \frac{4}{5}\rho_w \ kg/m^3 = \frac{4}{5} \times 10^3 = 800$$

#





$$T = mg\left(1 - \frac{\rho}{\sigma}\right)$$

T ↑ ↑ FB ↑ a
↓ mg

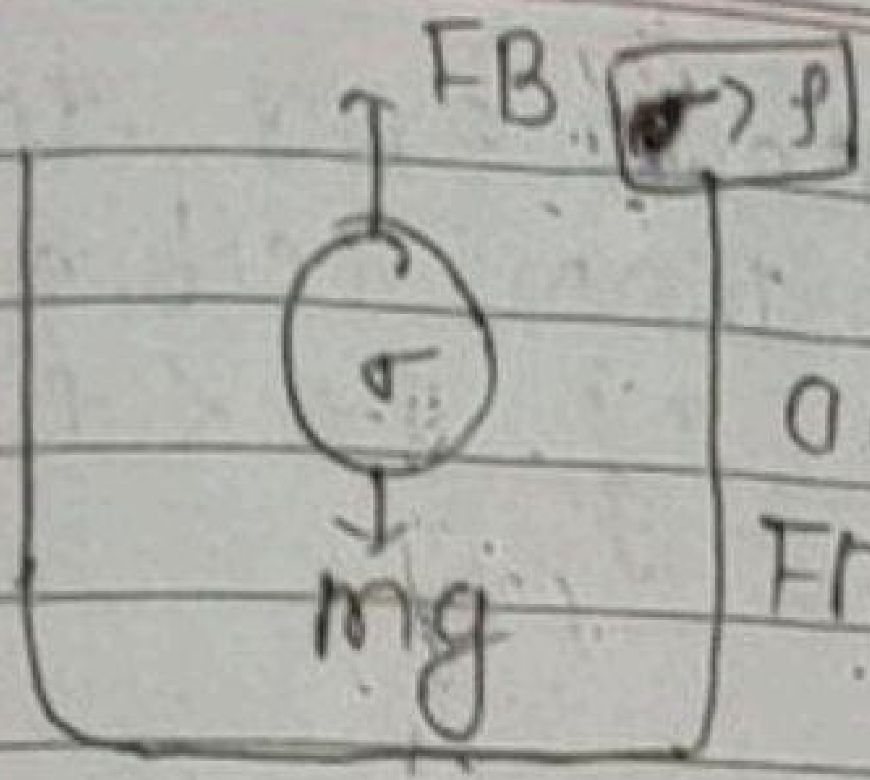
$$T + F_B = mg$$

$$T' = mg - F_B$$

$$\left[T' = mg\left(1 - \frac{\rho}{\sigma}\right)\right] \quad (1)$$

$$\left[\frac{T'}{T} = \frac{g+a}{g}\right]$$

$$T' = m(g+a)\left(1 - \frac{\rho}{\sigma}\right)$$



$$mg - F_B = ma$$
$$\sigma V g - \rho V g = ma$$

accn = ?

$F_{net} = ma$ $\sigma V g\left[1 - \frac{\rho}{\sigma}\right] = ma$

$$mg\left[1 - \frac{\rho}{\sigma}\right] = ma$$

$$a = g\left[1 - \frac{\rho}{\sigma}\right]$$

A piece of solid weighs 120 g in air, 80 g in water and 60 g in a liquid. The relative density of solid and that of liquid

$W = mg = 120g$ $N = mg\left(1 - \frac{\rho}{\sigma}\right)$

$N = 80g$ (water) $80g = 120g\left(1 - \frac{1}{\sigma}\right)$

$N = 60g$ (liquid

$\frac{2}{3} = 1 - \frac{1}{\sigma}$ $\frac{1}{\sigma} = 1 - \frac{2}{3} = \frac{1}{3}$

$\boxed{\sigma = 3}$ Solid

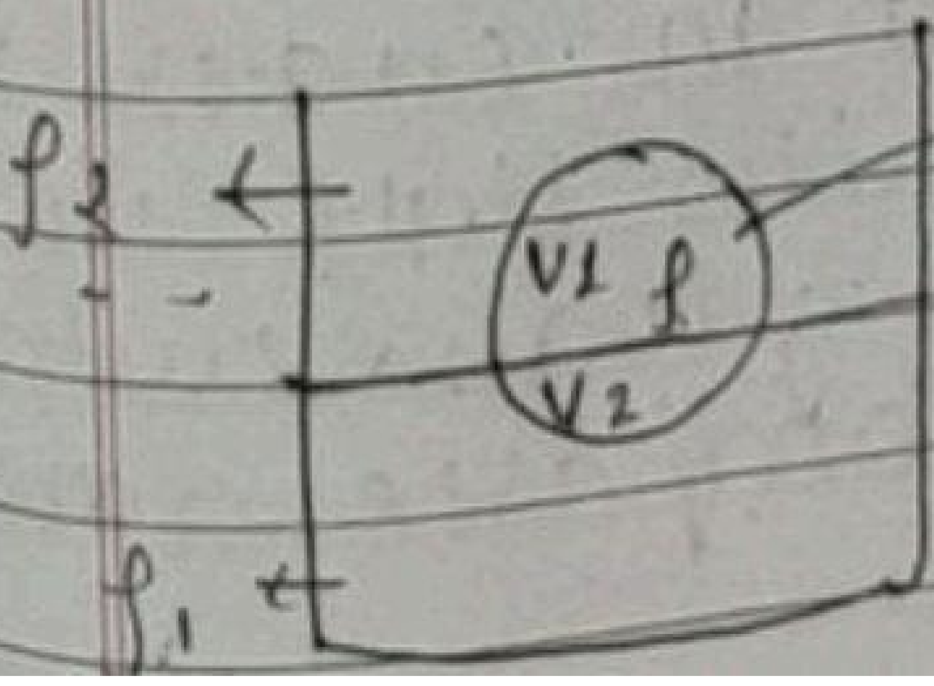
$N = mg\left(1 - \frac{\rho}{\sigma}\right)$

$60g = 120g\left(1 - \frac{\rho}{3}\right)$

$\frac{1}{2} = 1 - \frac{\rho}{3}$ $\frac{\rho}{3} = 1 - \frac{1}{2} = \frac{1}{2}$

$\boxed{\rho = 3/2}$ liquid

$V_T = V_1 + V_2$



→ floating in Eq. condition compare densities

$\rho_1 > \rho > \rho_2$

~~$\rho = \rho_1 + \rho_2$~~

$\rho = \frac{\rho_1 + \rho_2}{V_T = V_1 + V_2}$

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Q) A solid sphere of volume V and density ρ at the interface of two immiscible liq. of densities ρ_1 and ρ_2. If $\rho_1 < \rho < \rho_2$

$$mg = F_1 + F_2$$

$$\rho V g = \rho_1 V_1 g + \rho_2 V_2 g$$

$$\rho V_1 + \rho V_2 = \rho_1 V_1 + \rho_2 V_2$$

$$\rho V_1 - \rho_1 V_1 = \rho_2 V_2 - \rho V_2$$

$$V_1(\rho - \rho_1) = V_2(\rho_2 - \rho)$$

$$\frac{V_1}{V_2} = \left(\frac{\rho_2 - \rho}{\rho - \rho_1}\right)$$

Q) A metallic sphere weighing 3 kg in air is held by by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of metallic 10. The tension is-

$$T = mg\left(1 - \frac{\rho}{\sigma}\right) = 3 \times 10\left[1 - \frac{0.8 \times 10}{10 \times 10}\right]$$

$$30\left[1 - \frac{8}{100}\right] = 30 \times \left[\frac{92}{100}\right] \quad 30 \times 0.92$$

Q) Two liquid A and B float in water. It is observed that A floats with half its volume immersed and B floats with 2/3 of volume immersed. Compare density of A and B.

$\frac{V_{In}}{V_{out}} = \frac{1}{2} = \frac{\sigma A}{\rho_1}$ $\sigma A = \frac{1}{2}$ $\frac{\sigma A}{\sigma B} = \frac{1}{2} \times \frac{3}{2} = 3/4$

$\frac{V_{o In}}{V_{out}} = \frac{1}{3} = \frac{\sigma B}{\rho_1}$ $\sigma B = \frac{2}{3}$

Q) A body weight w in air and 0.8w in water. Density of material of the body is.

$mg = w$ $N = mg\left[1 - \frac{\rho}{\sigma}\right]$
$N = 0.8w$

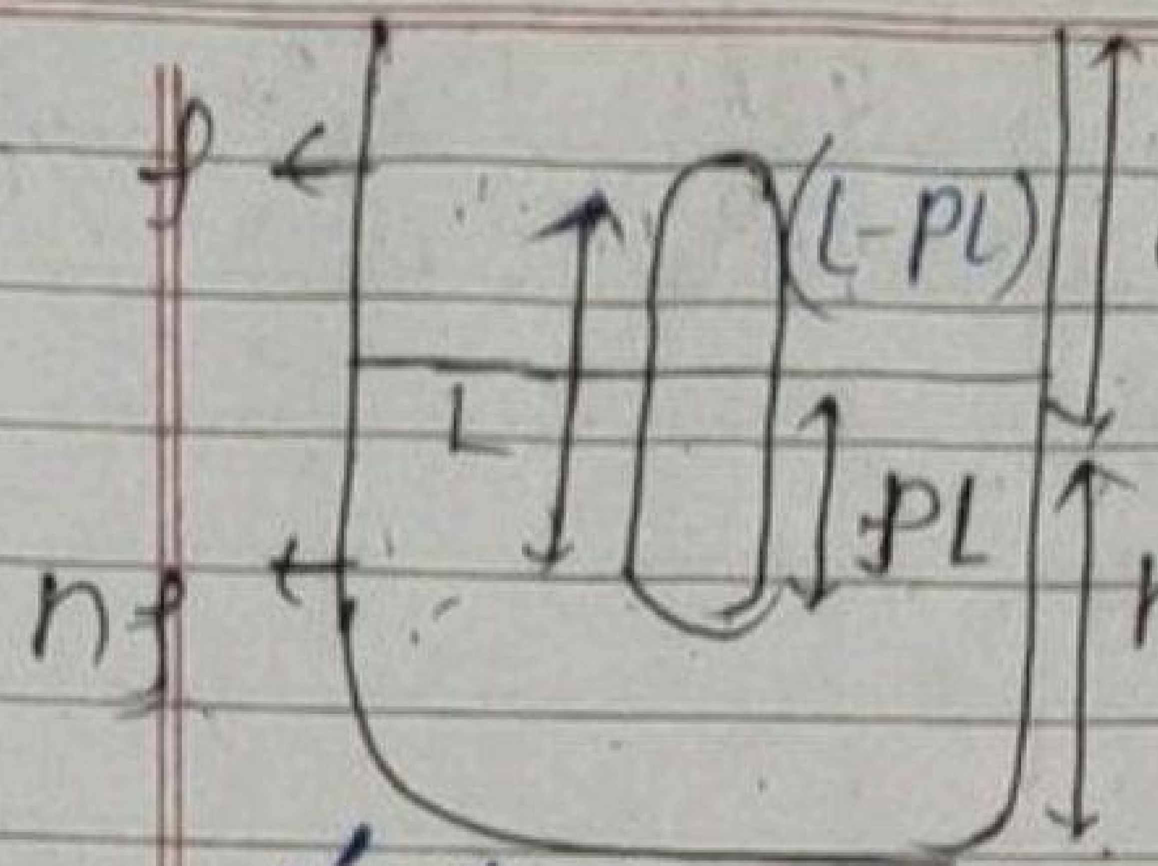
$\frac{0.8}{10} w = w\left[1 - \frac{\rho}{\sigma}\right]$

$\frac{\rho}{\sigma} = 1 - \frac{4}{5} = \frac{1}{5}$ $\sigma = 5\rho$
$= 5 \times 10^3 \, Kg/m^3$

Q) will an object of density ρ float in a fluid of density 1.5ρ? If yes, what fraction of the object's volume will remain above fluid?

$\frac{V_{In}}{V_{ow}} = \frac{\rho}{\frac{3}{2}\rho} = 2/3$ $\frac{V_I}{V_{In}} = 1 - \frac{V_I}{V_{ow}} = 1 - \frac{2}{3} = 1/3$

Buyant farce on a object with Cavity
Cavity से कुछ fark nahi padega.



$F_1 + F_2 = mg$

$\rho_1 V_1 g + \rho_2 V_2 g = \rho V_1 g$

~~$\rho A(L-PL)g + n\rho A PLg = \rho A L g$~~

$\rho(1-P) + n\rho P = d$

$\rho[1-P+np] = d$

$\rho\{1+p(n-1)\} = d$

Q) A piece of gold weighs 10 g in air and 9 g in water. What is the volume of cavity?

Volume of Cavity $= V_T - V_I(V_m)$

mass $= 10$ gm

$\rho V_m = 10$

$19.3 V_m = 10$

$V_I = V_m = \frac{10}{19.3} = 0.52$

$N = mg - F_B$

$9g = 10 - F_B$

$F_B = 1g$

$\rho V_T g = 1g$

$V_T = 1\ cm^3$

Buyont Hamesha total Volume ki wajah se lagega

$V_C = V_T - V_m$

$= 1 - 0.052 = 0.482\ cg$

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Q) An Ice cube is floating in a water when Ice melts then water level will

float apparent wt = 0
$F_B = mg$
weight of displaced fluid = weight of Ice

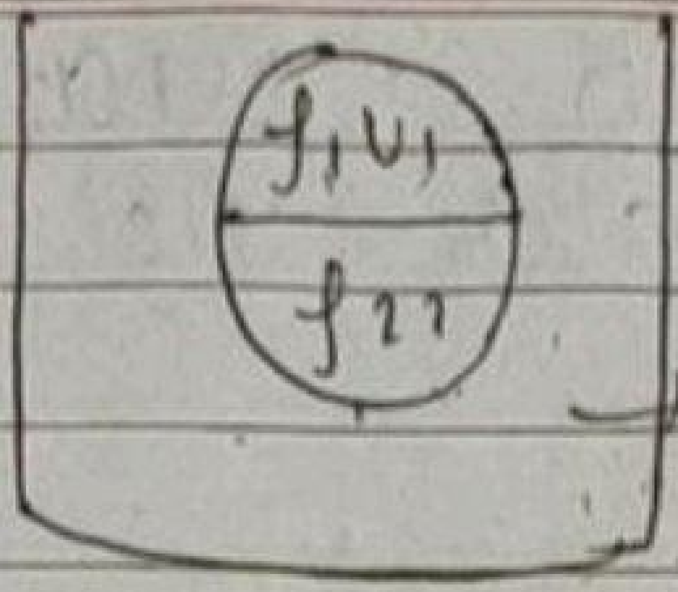
weight of diplaced water = weight of water formed by Ice

A level remain same

Q) A Block of ice floats on in on oil in a vessel when the ice melts. the level of oil will.
float
$F_B = mg$
weight of displaced liq. = weight of Ice
weight of oil = weight of water

$\downarrow \rho_o V \uparrow$ $\uparrow \rho_w V_w \downarrow$
V_w कम हो जाए तो oil neeche jaega use bharne

A $\rho_{oil} < \rho_w$



$$F_1 + F_2 = mg$$

$$\rho_1 V_1 g + \rho_2 V_2 g = \rho V_T g$$

$$\rho_1 V_1 + \rho_2 V_2 = \rho V_1 + \rho V_2$$

$$\rho_1 V_1 - \rho V_1 = \rho V_2 - \rho_2 V_2$$

$$V_1(\rho_1 - \rho) = V_2(\rho - \rho_2)$$

$$\left[\frac{V_1}{V_2} = \frac{\rho - \rho_2}{\rho_1 - \rho} \right]$$

Fluid Dynamics

→ fluid must be Ideal
→ frictional less (non-viscous)
→ Incompressible
→ Irrotational

Streamline flow

$V_1 = V_2 = V_3 = V_4$ not necessary

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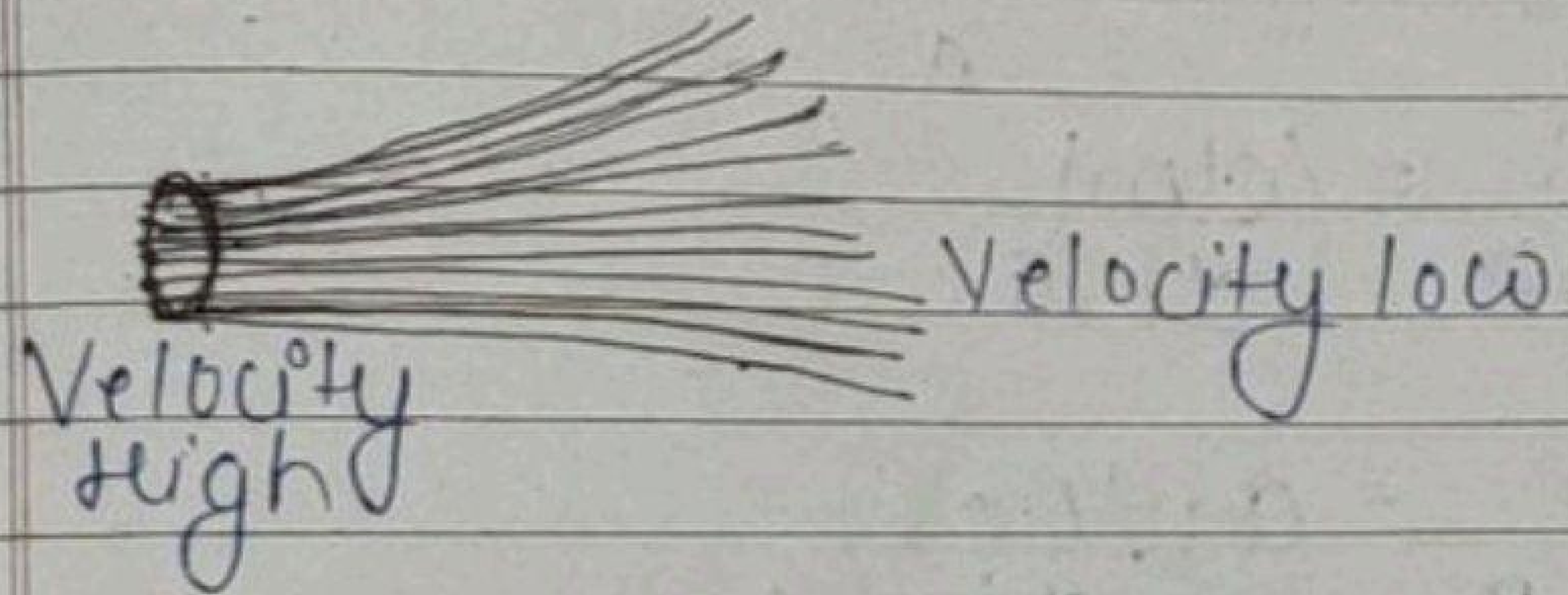
→ When a liquid flows such that each particles of the liquid passing a given point moves along the same path and has the same velocity as its predecessor had at that point, the flow is called Stremline



Properties of Streamlines

① Tangent at any point gives of flow of liquid

② no two streamline can cross each other

③ fluid velocity remains constant at any point of a streamline, but it may be different at different points of same streamline

④ Streamline close greater fluid velocity

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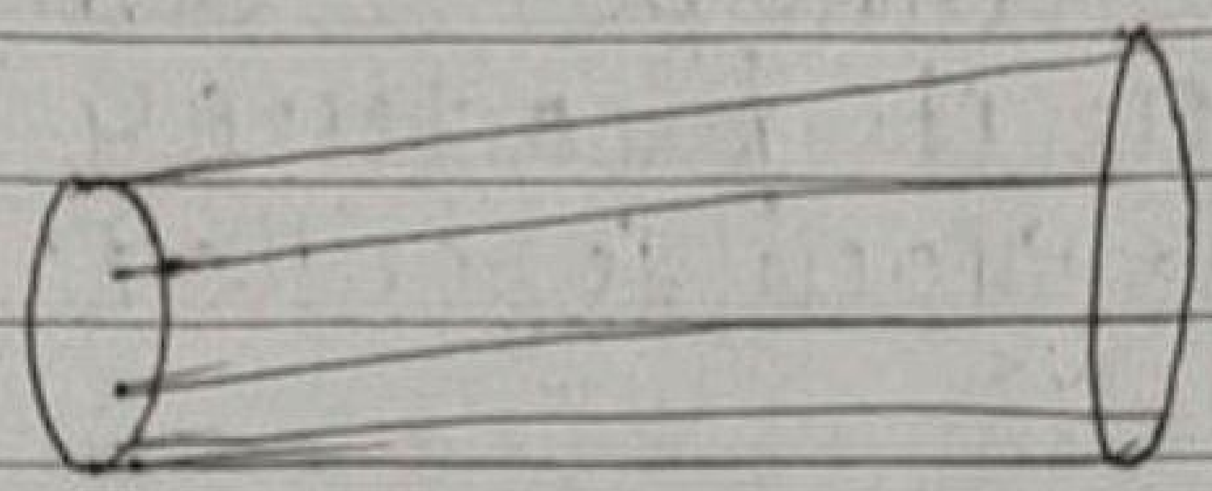
Equation of continuity

→ Based on conservation of mass


P Q

$$\left(\frac{dm}{dt}\right)_P = \left(\frac{dm}{dt}\right)_Q$$

$$\frac{d(\rho A_P l)}{dt}_P = \frac{d(\rho A_Q l)}{dt}_Q$$

$$A_P\left(\frac{dl}{dt}\right)_P \qquad A_Q\left(\frac{dl}{dt}\right)_Q$$

$$A_P V_P = A_Q V_Q$$

rate of volume flow = constant

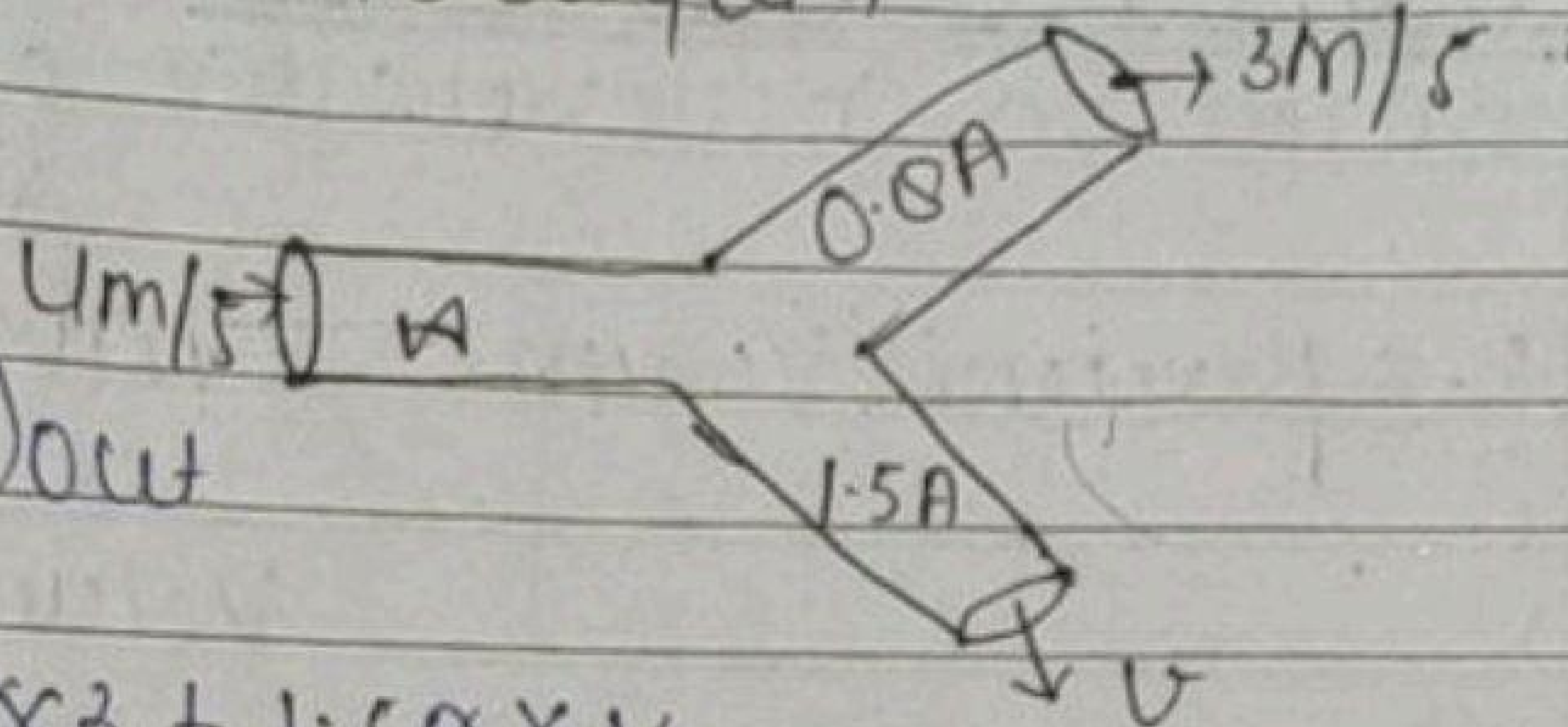
$$(AV)_{Incoming} = (AV)_{outgoing}$$

Q) A liquid flows in the tube from left to right. A_1 and A_2 are the cross section of the portion of tube. the ratio of $\frac{V_1}{V_2}$

$$A_1 V_1 = A_2 V_2$$

$$\frac{V_1}{V_2} = \frac{A_2}{A_1}$$

An Incompressible liquid



$(AV)_{In} = (AV)_{out}$

$A \times 4 = 0.8A \times 3 + 1.5A \times v$

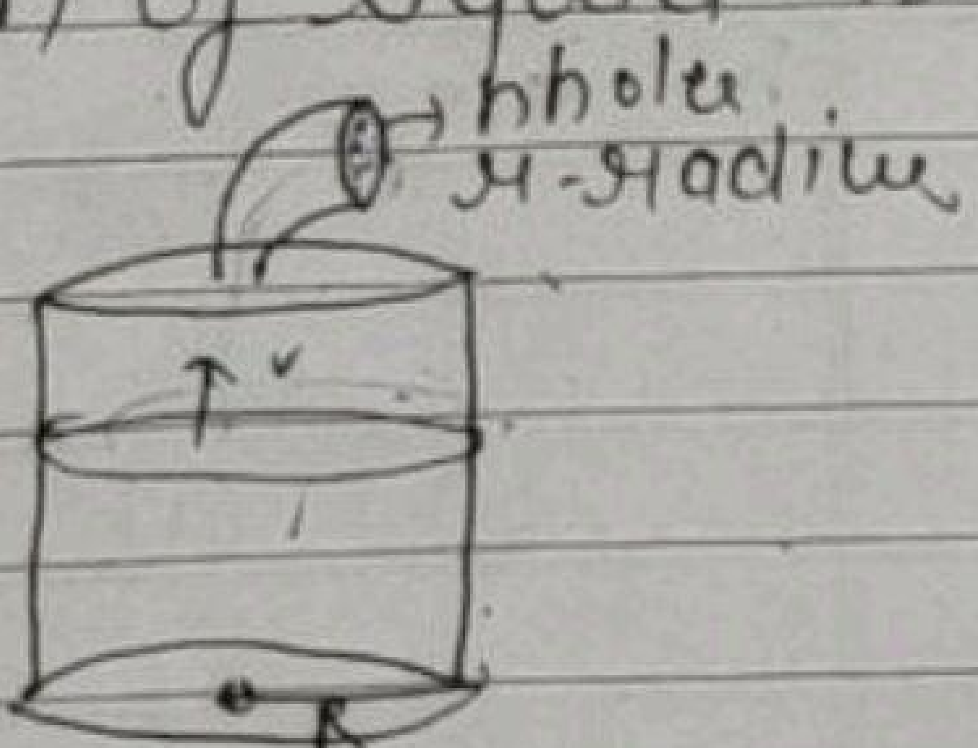
$4 = 2.4 + 1.5v$

$4 - 2.4 = 1.5v$

$1.6 = 1.5v$

$v = \frac{1.6}{1.5} = 1.1\ m/s$

Q) The cylindrical tube of a spray pump has radius R, one end of which has n fine Holes, each of radius r. if the speed of liquid in the tube is V, the speed of the ejection of liquid through the Holes is



$A_1 V_1 = A_2 V_2$

$\pi R^2 V = (\pi r^2 V_0) n$

$\left[V_0 = \frac{V R^2}{r^2 n} \right]$

Q)

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Bernoulli's Equation

Based on conservation of energy

K.E per unit Volume $= \frac{\frac{1}{2} mv^2}{Volume} = \frac{1}{2}\rho v^2$

P.E per unit Volume $= \frac{mgh}{V} = \rho gh$

$P + \frac{\rho}{2} v^2 + \rho gh = \text{Constant}$

$P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$

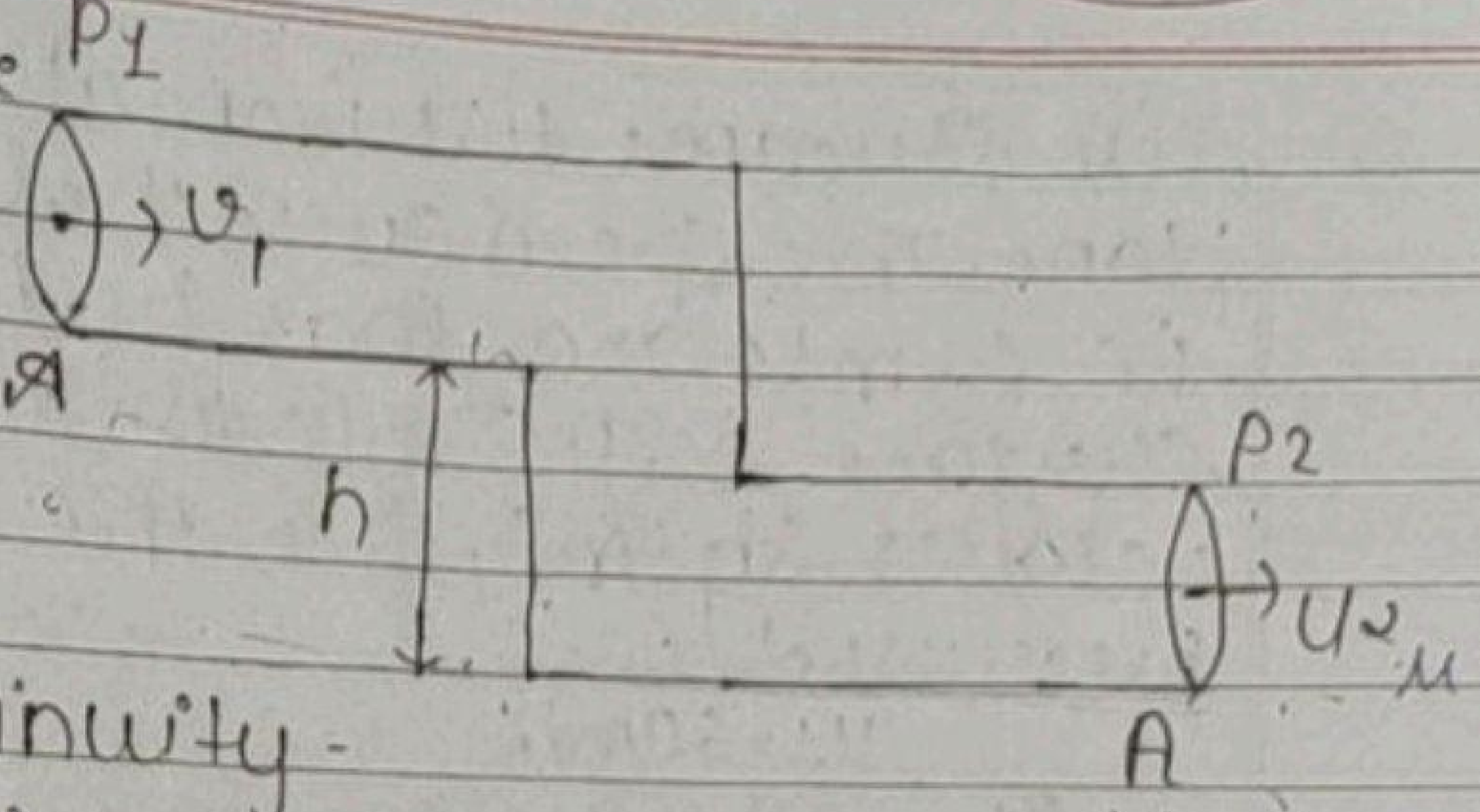
$\rho A - P$

P – alag

v – alag

h – alag

a) Area = same



Eqn of continuity -

$A v_1 = A v_2$

$v_1 = v_2 = v$

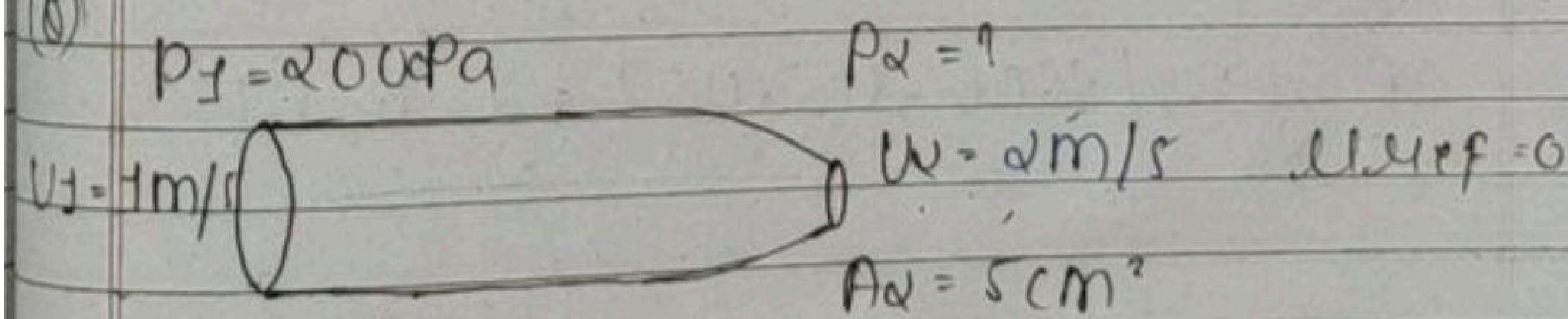
Bernolli Equation -

$P_1 + \rho g h + \frac{1}{2}\rho v^2 = P_2 + 0 + \frac{1}{2}\rho v^2$

$P_1 + \rho g h = P_2$

$[P_1 < P_2]$

(b)



$P_1 + \frac{1}{2}\rho v_1^2 + 0 = P_2 + \frac{1}{2}\rho v_2^2 + 0$

$2000 + \frac{1}{2} 10^3 (1)^2 = P_2 + \frac{1}{2} 10^3 (2)^2$

$2000 + \frac{1}{2} 10^3 = P_2 + 2 \times 10^3$

$2000 + 500 = P + 2000$

$P = 500 Pa$

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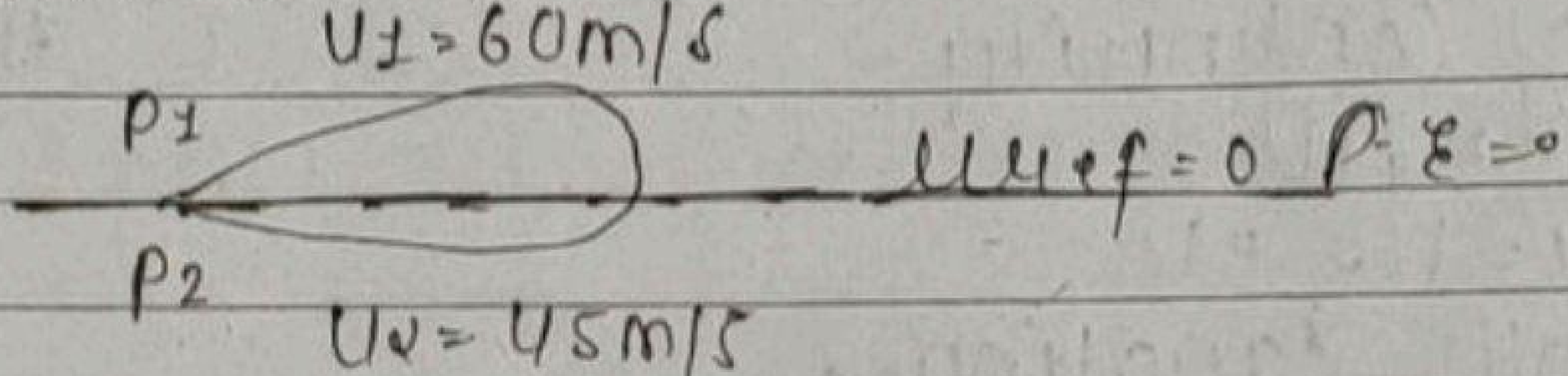
Q) Air streams horizontally past an air plane. The speed over the top surface is 60 m/s and that under the bottom surface is 45 m/s. the density of air is 1.293 kg/m³, then the difference in pressure is.



$$P_1 + \frac{1}{2}\rho U_1^2 + 0 = P_2 + \frac{1}{2}\rho U_2^2 + 0$$

$$\frac{1}{2}\rho U_1^2 - \frac{1}{2}\rho U_2^2 = P_2 - P_1$$

$$\frac{1}{2}\rho(U_1^2 - U_2^2) = \Delta P$$

$$\Delta P = \frac{1}{2} 1.3[3600 - 45\times45] = \frac{1}{2}\times 1.3\times 1575 = 1023$$

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{Const}^n$$

Divided by ρg both side

$$\left[\frac{P}{\rho g}\right] + \left[\frac{v^2}{2g}\right] + h = \frac{\text{Const}^n}{\rho g}$$

↓ Pressure Head of liq. ↳ Velocity Head ↳ gravitational Head

Each Head dimension equal to dim^n of length.

Q) At what speed, the velocity Head of water is equal to pressure Head of 40 cm of mercury.

Velocity Head of water $= \frac{v^2}{2g}$

Pressure Head of water $= \frac{P}{\rho_w g}$

$\frac{v^2}{2g} = \frac{P}{\rho_w g}$ $v^2 = \frac{2P}{\rho_w} = \frac{2 \rho_{Hg} 40\,cm\, g}{\rho_w}$

$v^2 = 2 \times 13.6 \times 40 \times 10^{-2}$

$v = 13.6 \times 8 = 104$

$v \quad \sqrt{104} = 10.3$

Q) Pressure at a point is 80 cm of water

$P = \rho_w g H$

$10^3 \times 10 \times 80 \times 10^{-2}$

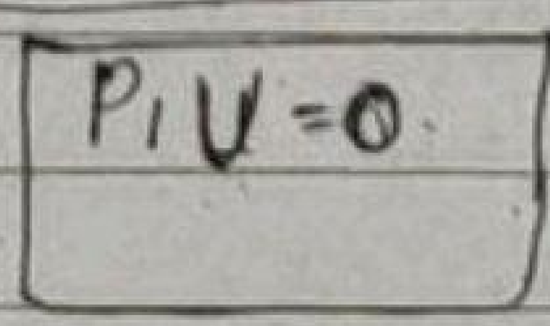
$8 \times 10^3 = 8000\ Nm^2$

Q) A wind with speed 40 m/s blows parallel to the roof of a House. The area of the roof is 250 m^2. Assuming that the pressure inside the House is atm. p, the force exerted by the wind on the roo[illegible] and the direction of force will be

$P_{air} = 1.2\ kg/m^3$

$P_2 = 1.2 \quad v_2 = 40\ m/s$

$w_{ref} = 0$



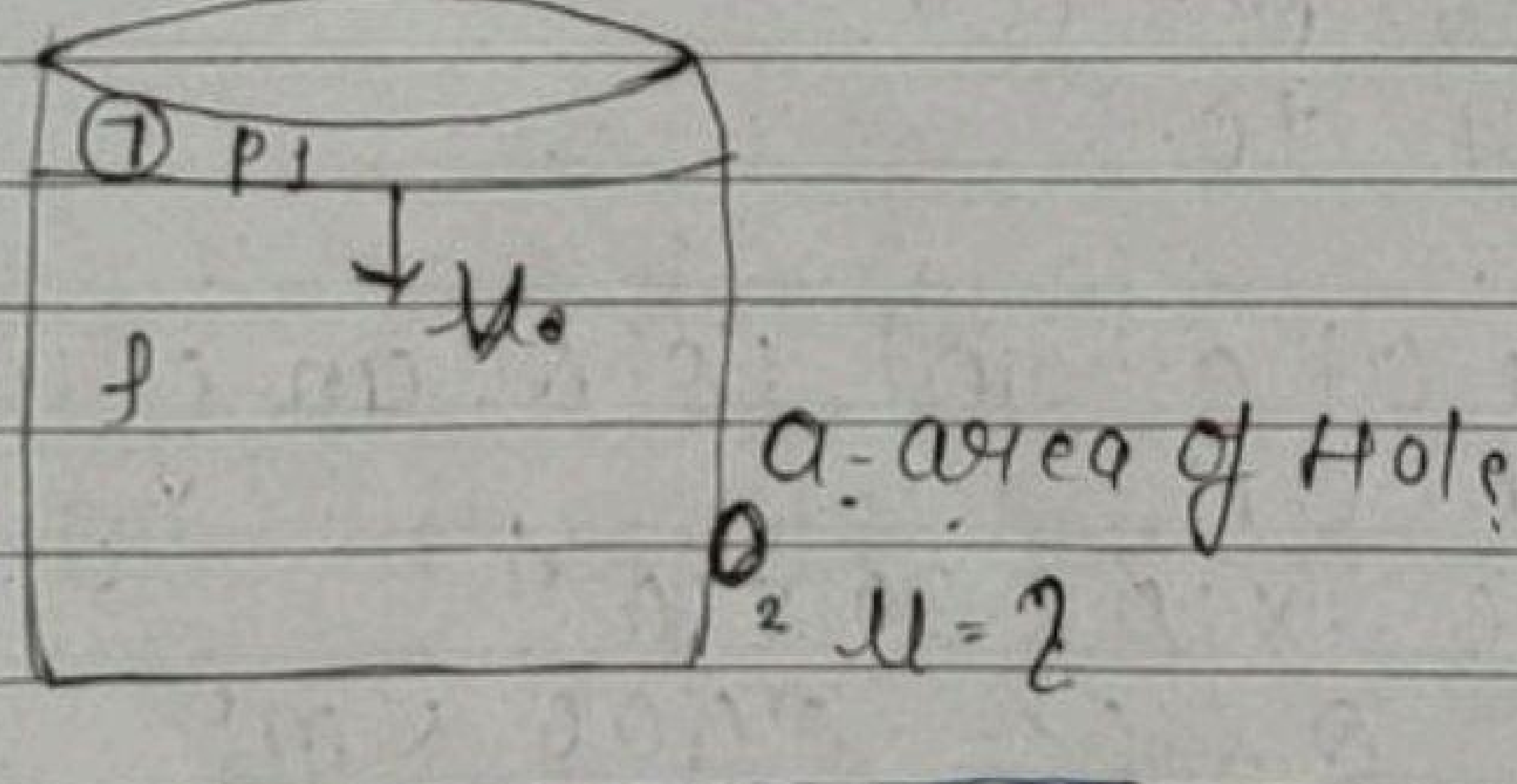
$P_1 + 0 + 0 = P_2 + 0 + \frac{1}{2}\rho V^2$

$A \times P_1 - P_2 = \frac{1}{2}\rho v^2 \times A^2$

$F_{upward} = \frac{1}{2} \times 1.2 \times 40 \times 40 \times 2 \times 50$

$= 1.2 \times 5000 \times 40 = 2.4 \times 10^5\ N$ upward

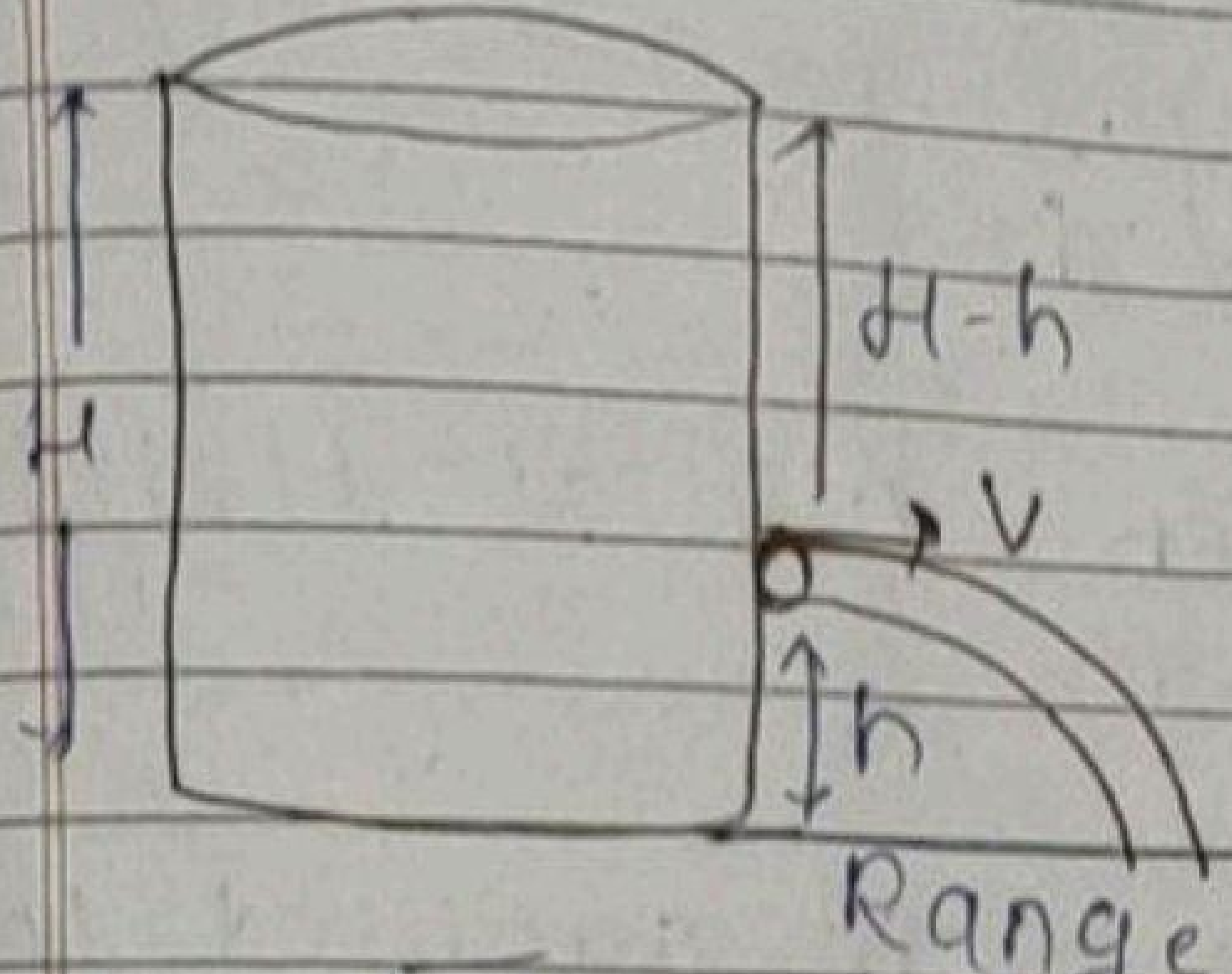
✡ Velocity of Efflux -



$$V = \sqrt{\frac{2(P_1 - P_0)}{\rho} + 2gh}$$

→ Torricelli chacha assumed Container is open $V = \sqrt{2gh}$

Find Velocity of efflux, Range, Time of flight



Velocity = $\sqrt{2g(H-h)}$

Range = $u_x \times T_f$

$= 2\sqrt{(H-h)h}$

$T_f = \sqrt{\frac{2h}{g}}$

Find R max.

R will be max at $h = H/2$

$R_{max} = 2\sqrt{h(H-h)}$

$2\sqrt{H/2\,(H-H/2)}$ $\quad 2\sqrt{\frac{H^2}{4}}$

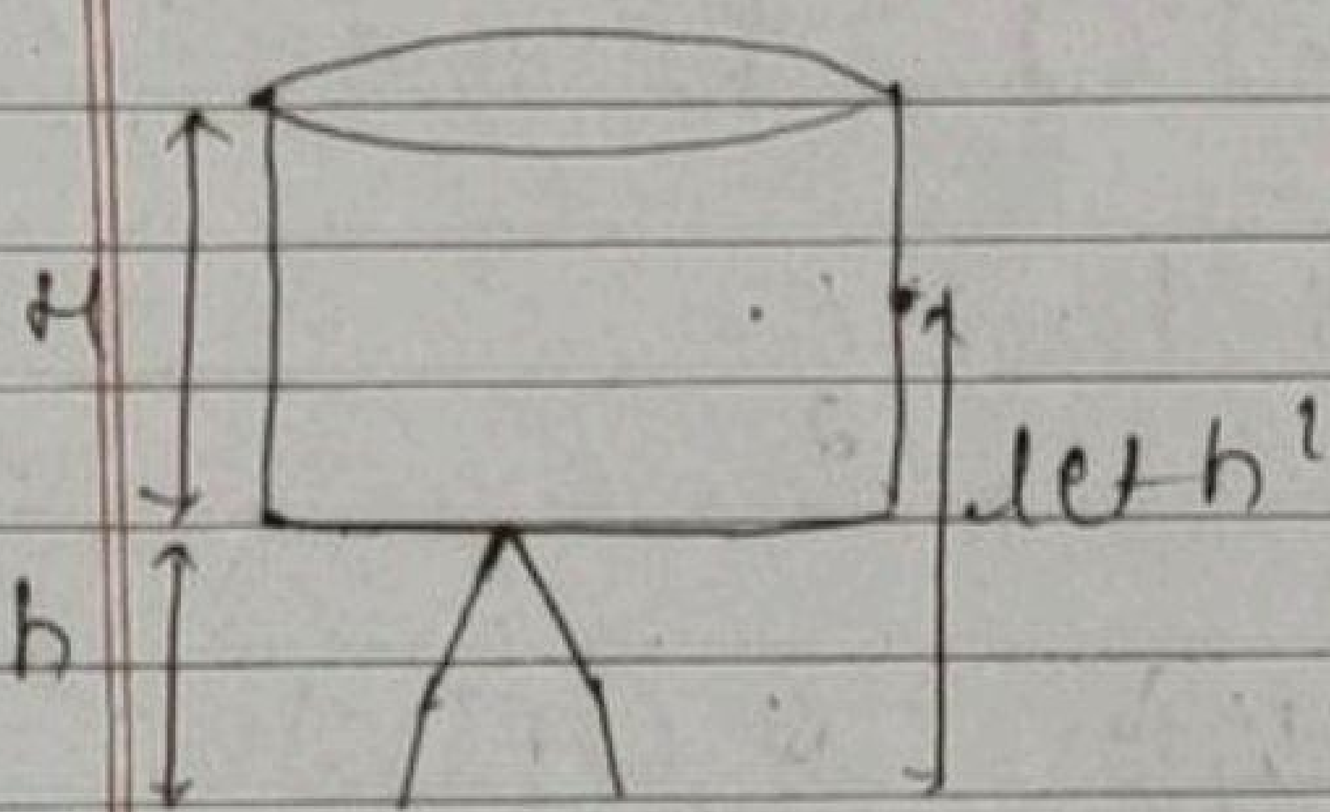
$\frac{2}{2}\sqrt{H^2}$

$R_{max} = H$

A tank is filled with water and two Holes A and B are made in it. for getting same range ratio of $\frac{h}{h'}$ is

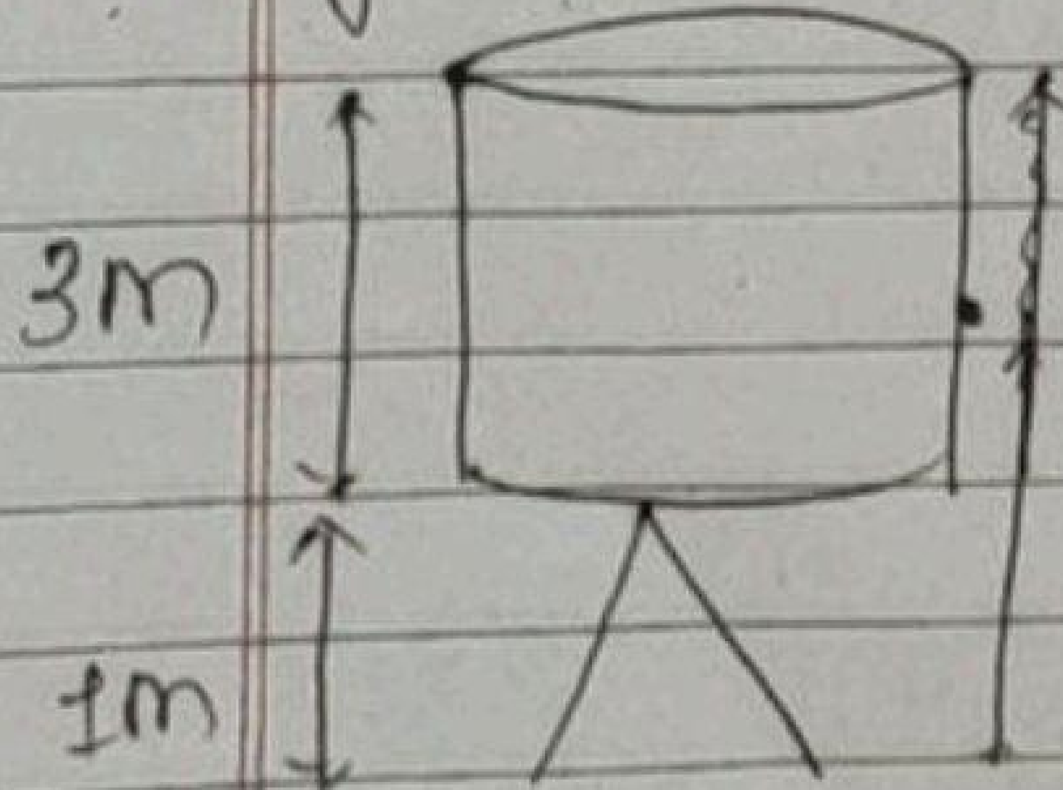
If range is same then ratio of Height is 1.

Q) find position of Hole from ground for maximum range.



$h' = \left(\frac{H+h}{2}\right)$ max Range

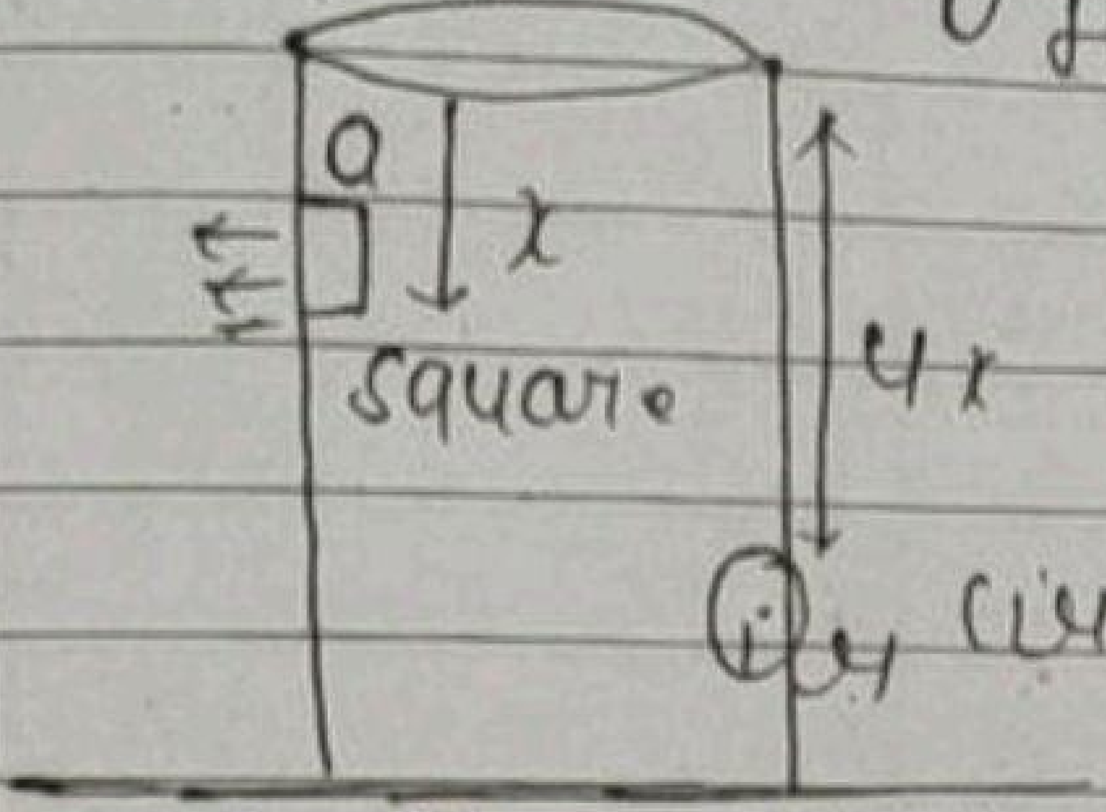
Q) water is filled, what will be the Height of a Hole made in it, so that water came to be sprayed upto max Horizontal distance on ground?



Height $= 3+1 = 4/2 = 2m$ from ground

$h' = \frac{H+h}{2} =$

A large open tank has two Holes, water flowing out per second from both Holes are same the r is equal to



$\frac{dm}{dt}$ = rate of quantity per sec

$= \rho A V$ ↓ same

$A_1 V_1 = A_2 V_2$

$a^2 \sqrt{2gx} = \pi r^2 \sqrt{2g4x}$

$a^2 = \pi r^2 2 \quad r^2 = \frac{a^2}{2\pi} \quad r = \frac{a}{\sqrt{2\pi}}$

Q) If container is massless and placed on rough surface then find min^m coefficient of friction of surface so that container will not slide

Force on Container = $\rho a 2gH$

$\rho a 2gH = \mu N$

$\rho a 2gH = \mu mg$

$\rho a 2gH = \mu \rho HA g$

$\mu = \frac{2a}{A}$

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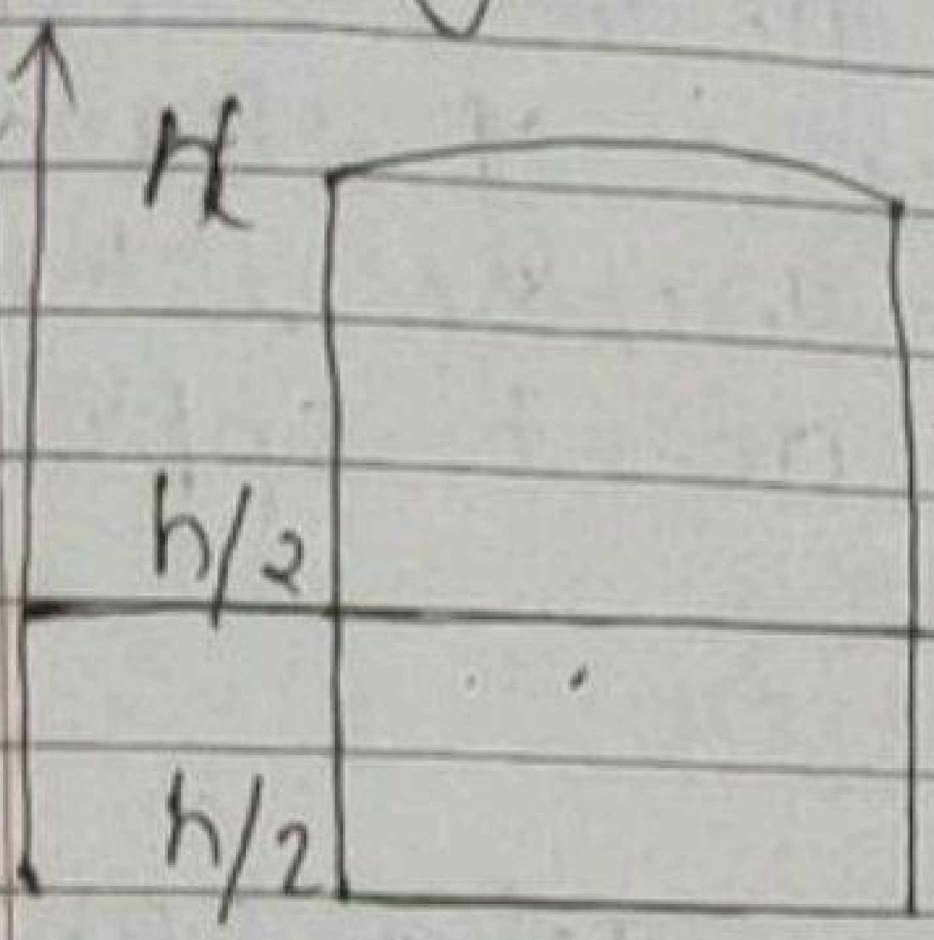
Q) A large tank filled with water



If T = total time taken H to Bottom

$t_1 = T - \frac{T}{\sqrt{2}}$

$t_2 = T/\sqrt{2}$

ratio of $t_1 : t_2$

$\frac{t_1}{t_2} = \frac{\sqrt{2}}{1} = \sqrt{2} : 1$

Q) A rectangular Vessel when full of water to a Height, takes 10 min to be emptied through an orifice in its bottom. How much time will it take to be emptied when Half filled with water.

$T = 10 \text{ min}$

$t = \frac{T}{\sqrt{2}} = \frac{10}{\sqrt{2}} = \frac{10}{1.41} = 7 \text{ min}$

Q) In the show Venturimeter fluid Velocities at two points are Indicated what is the difference in the levels of liq. at in the tube

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Viscosity:

when fluid flows then there a viscous friction b/w parallel layer of fluid due to relative motion b/w layers like

→ friction b/w two solid acts to oppose relative motion.

→ due to cohesive force.

$$\text{Fluid friction (viscous force)} = \eta A \frac{\Delta v}{d}$$

(η → depend on nature of liquid)

Difference b/w viscosity and solid friction

	Viscosity	Solid friction
(1)	directly proportional to the surface area of contact of fluid layers	Independent
(2)	directly proportional to relative velocity	Independent
(3)	doesn't depend on the normal reaction	directly proportional to normal rxn
(4)	depend on size and shape of solid	Independent of shape and size

Viscosity

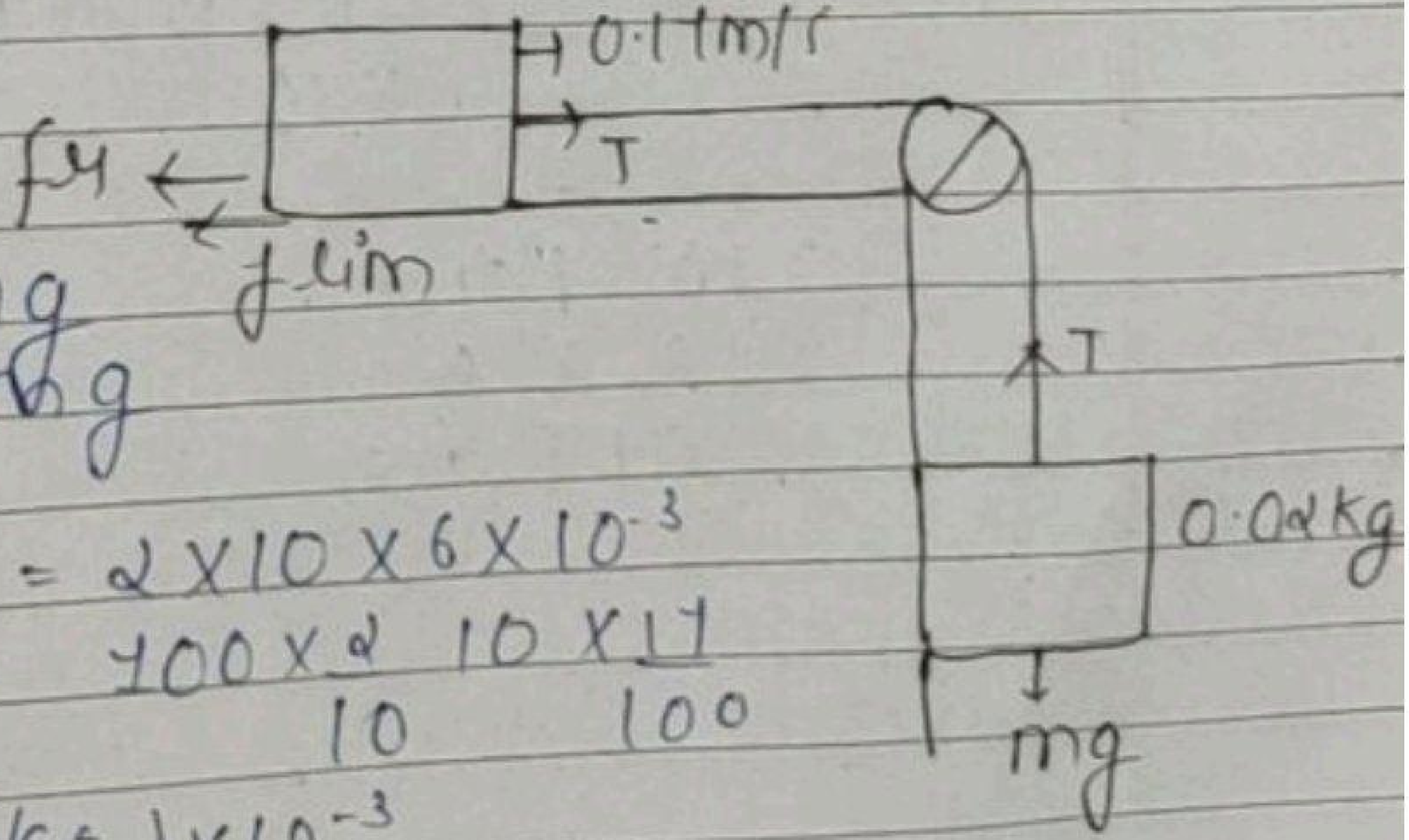
- It is an internal friction force b/w two layer of fluid only when there is a relative motion.
- directly proportional to relative velocity area of layer.
- acts because of cohesive

Q) A layer of glycerine of thickness 3mm is b/w a large surface area and a surface area of $0.2m^2$; with which force the small surface is to be pulled so, that it can move with a velocity of 5 m/s $\eta = 0.009$

$$F = \eta A \frac{\Delta v}{d} = \frac{9}{1000} \times \frac{2}{10} \times \frac{5}{3 \times 10^{-3}} = 60N$$

$\because 1mm = 10^{-3}m$

Q) Area - $0.2m^2$
$d = 0.6mm$
find η



$F_{lim} = T = mg$

$$\eta A \frac{\Delta v}{d} = mg$$

$$\eta = \frac{mgd}{A \Delta v} = \frac{2 \times 10 \times 6 \times 10^{-3}}{100 \times \frac{2}{10} \quad 10 \times \frac{11}{100}}$$

$$= \left(\frac{60}{11}\right) \times 10^{-3}$$

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Coefficient of viscosity - F/A

The ratio of shearing stress / tangential stress and strain rate. $\left(\frac{F/A}{V/l}\right)$ $\left(\frac{V}{l}\right)$

$\frac{F}{A} \times \frac{l}{V}$

η

unit = PI

1 PI = 10 Poise (CGS) Poise cm

Q) A metal plate of area $10^3\ cm^2$ rests on a layer of oil 6mm thick. A tangential force 10^{-2} N is applied on it to move it with a Horizontal / Constant Velocity of $6\ cms^{-1}$. The coefficient of viscosity of liquid is

$F_r = \eta A \frac{\Delta v}{d}$

$10^{-2} = \eta \times 10^{3} \times 10^{-1} \times \frac{6 \times 10^{-2}}{6 \times 10^{-3}}$

$\eta = 10^{-2}$ Poise

$10^{-2} \times 10$

0.1 Poise

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When a sphere move in a ~~solid~~ fluid (liquid/gas) then there a fluid friction acts on sphere oppose motion of sphere in fluid.

only for solid sphere

$F_r \propto$ Velocity

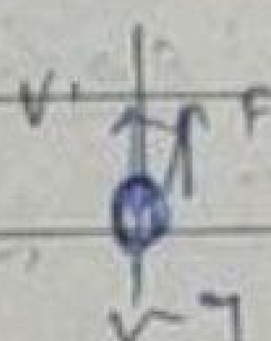
$\propto r$

$\propto \eta$

{ $\vec{F_r} = -6\pi r \eta \vec{v}$ Strokes law }

Terminal Velocity -

The max. constant velocity acquired by a body while falling through a viscous medium is called its terminal velocity. denoted by V_T.



$V_T = \dfrac{2r^2(\sigma - \rho)g}{9\eta}$

weight = upthrust + viscous drag

Q) A small hole of area of cross-section 2 mm² is cut near the surface bottom of a fully filled tank of height 2m. taking g = 10 the rate of flow of water through the open hole would be dem early.

V at bottom $= \sqrt{2gh} = 6.1$ m/s

$AV = 2 \times (10^{-3})^2 \times 6.1 = 12.2 \times 10^{-6}$

Flow =

Fluid friction b/w two layer of fluid

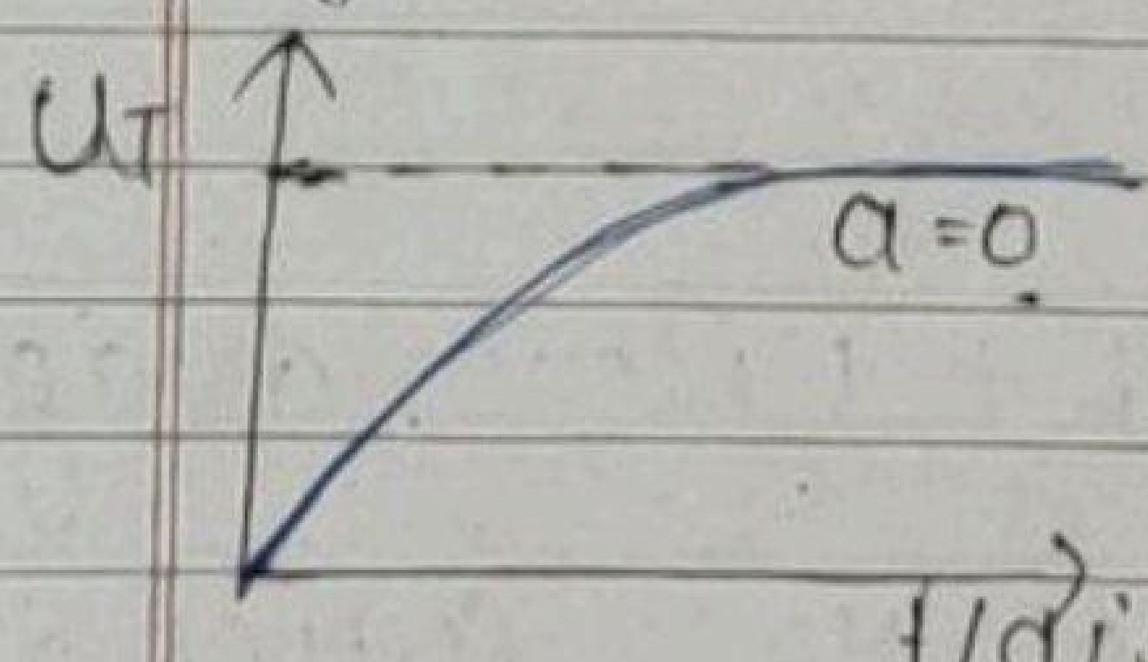
$F_r = \eta A \frac{\Delta v}{d}$

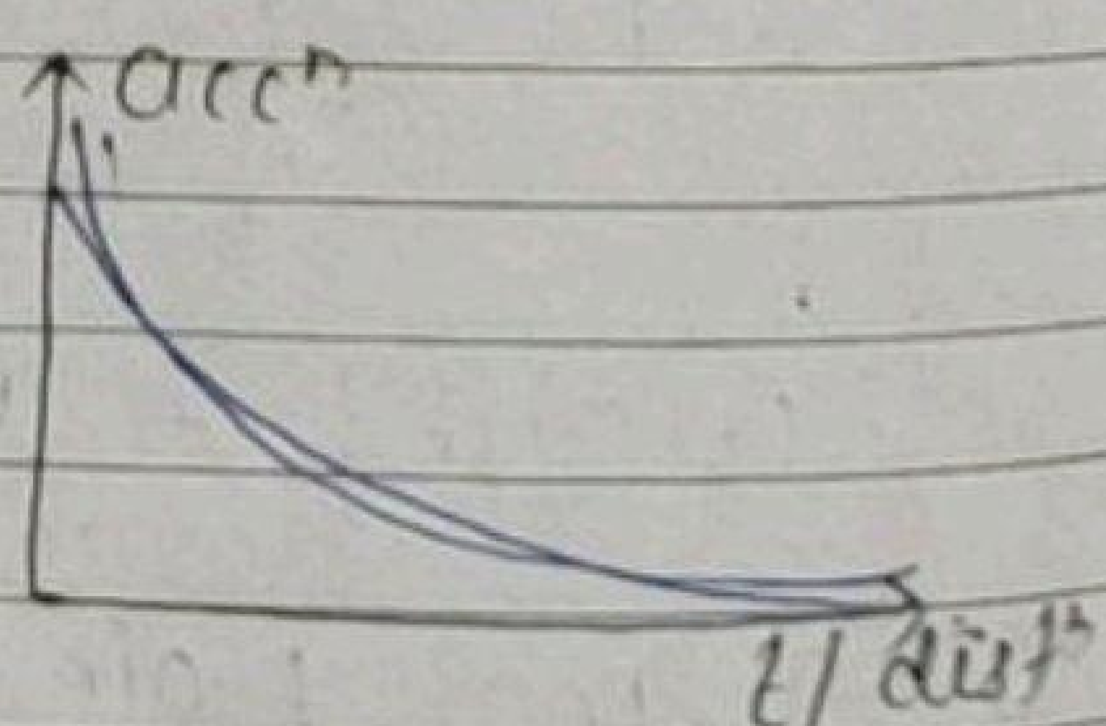
- $\eta \propto \frac{1}{temp}$ (liquid) cohesion force ↓ due to ↑ in kinetic energy
- $\eta \propto temp$ (gas) ↑ in the rate of diffusion with rise in the temp.

Fluid friction b/w solid and fluid

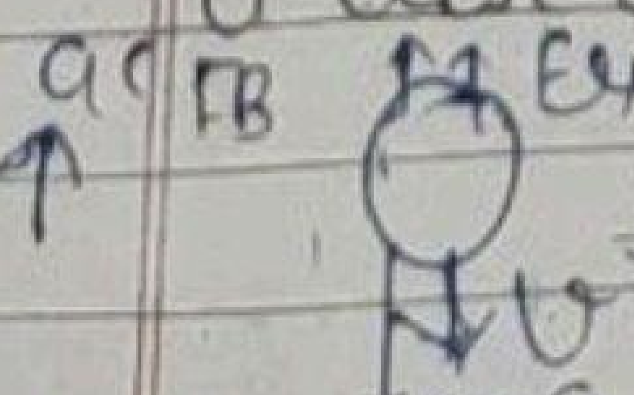
$F_r = 6\pi\eta r \vec{v}$

- Velocity jidhar uske opposite fluid friction
- Neeche jaker velocity constant ho jati hai agar initially 0 ho to.



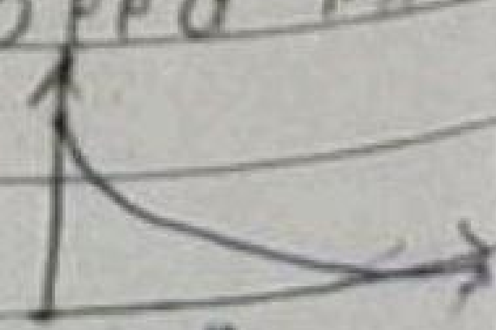


Q) A ball is thrown downward with some velocity, greater than terminal speed into a viscous fluid. draw curve.



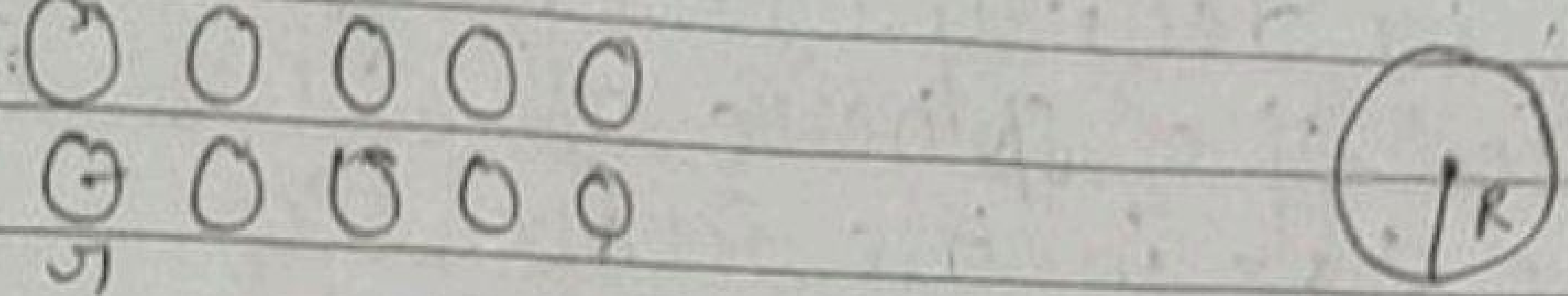
$v > v_T$ (t=0)

$(F_B + F_r) > mg$ retardation



$u \downarrow$

n-water drops comes to form a single drop then its terminal Velocity will be



$V_{Ti} \propto r^2$ $V_{Tf} \propto R^2$

$[R = n^{1/3} r]$

$$V_{Tf} = V_{Ti}\, n^{2/3}$$

Q). A small drop of water falls from rest through a large Height h in air, the final velocity is

Almost Independent to h.

Q) A ball of density σ and radius r is dropped on the surface of a liq. of density ρ from certain Height. If speed of ball doesn't change even on entering in liq. and Viscosity of liquid is η then the Height is.

$$V_T = \sqrt{2gh} = \frac{2r^2(\sigma - \rho)g}{9\eta}$$

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Q) If the terminal speed of a sphere of gold ($\rho = 19.5$) is 0.2 m/s in a liquid density ($\rho = 1.5$) find the terminal speed of a sphere of silver ($\rho = 10.5$) of same size in the same liq.

r same
η same

$V_T = (\sigma - \rho)$

$$\frac{0.2}{V_T} = \frac{(19.5 - 1.5)}{(10.5 - 1.5)} = \frac{18}{9} = 2$$

$V_T = 0.1$

Q) A small sphere of radius r falls from rest in a viscous liquid. As a result Heat is produced due to viscous force. The rate of production of Heat which the sphere attain its terminal velocity prop. to.

$$\frac{dH}{dt} = \text{Power} = F \cdot V$$

$6\pi\eta r v \times v$

$r v^2$ $\quad \because v \propto r^2$

$r(r^2)^2$

$r \times r^4 = r^5$

Poiseullie Equation: The rate of volume flow of viscous liquid through pipe.

$$Q = \frac{dV}{dt} = AV \quad \propto \Delta P = P_1 - P_2$$

$Q \propto r^4$

$Q \propto \frac{1}{l}$

$Q \propto \frac{1}{\eta}$

$$Q = \frac{\pi \Delta P r^4}{8 \eta l} = \frac{\Delta P}{\pi r^4} = \frac{\Delta P}{\left(\frac{8\eta l}{\pi r^4}\right)} = \frac{\Delta P}{R}$$

R- fluid resistance

$\propto l$

$\propto \frac{1}{r^4}$

Combination of Pipe

Series Combination of Pipe

$$Q_1 = Q_2 = Q = \frac{\Delta P}{R} = \frac{P_1 - P_2}{Req}$$

$$\frac{P_1 - P_2}{Req} = \frac{\Delta P_1 + \Delta P_2}{R_1 + R_2} = \frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}$$

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Parallel Combination

ΔP - Same

$[Q_{net} = Q_1 + Q_2]$

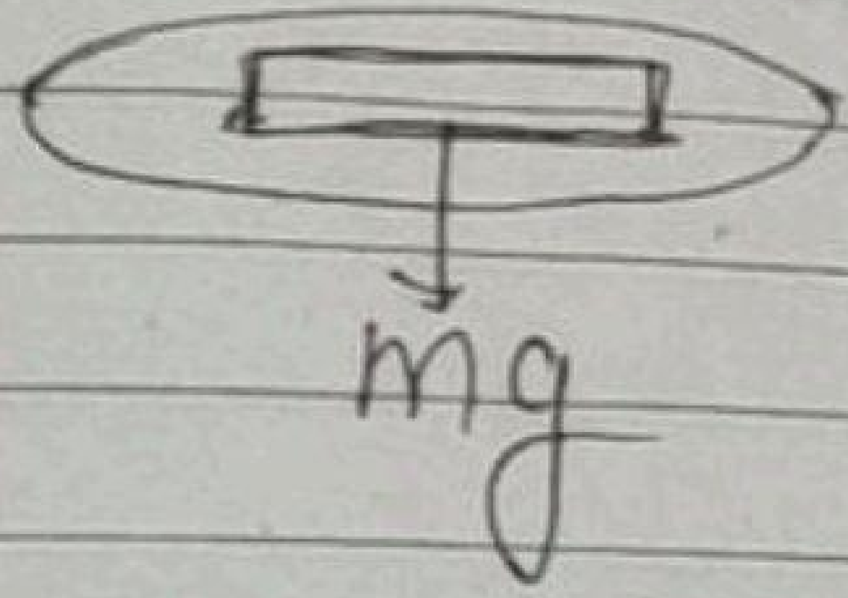
Surface tension-

→ It is a property of static fluid only not the property of gas and flowing liquid

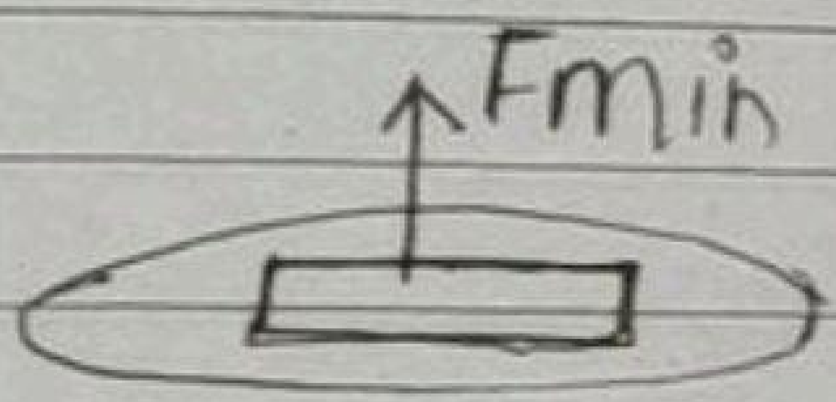
→ due to cohesive force

→ property of free surface of liquid

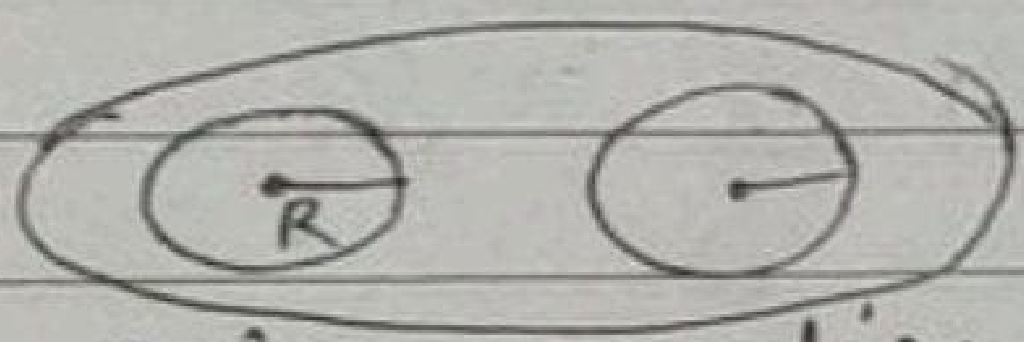
$S = \frac{F}{l}$ Force $= Sl$



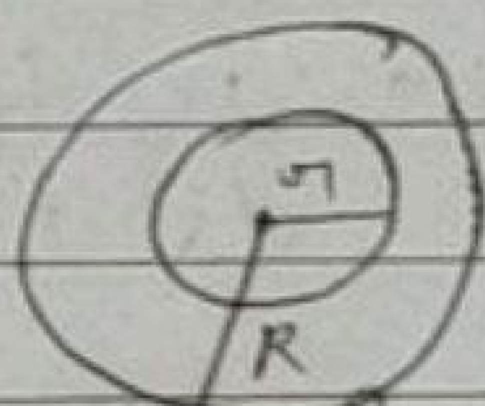
$F = S(2l) = mg$



$F = S(2l)$



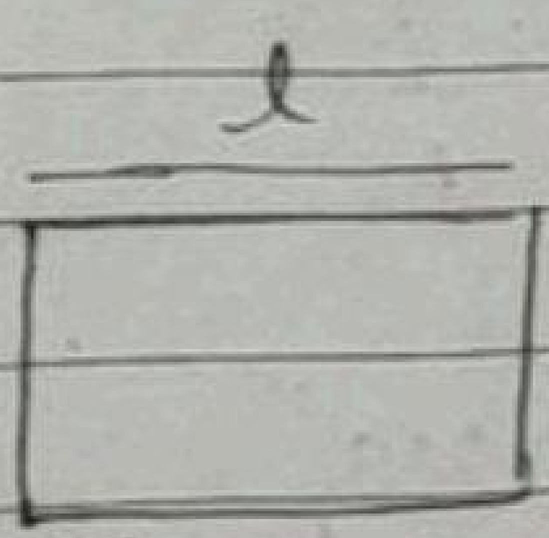
ring disc



$F = S2\pi R \times 2$ $F = S2\pi R$ $F = S(2\pi R + 2\pi r)$

$S4\pi R$

$F = S(4l)$

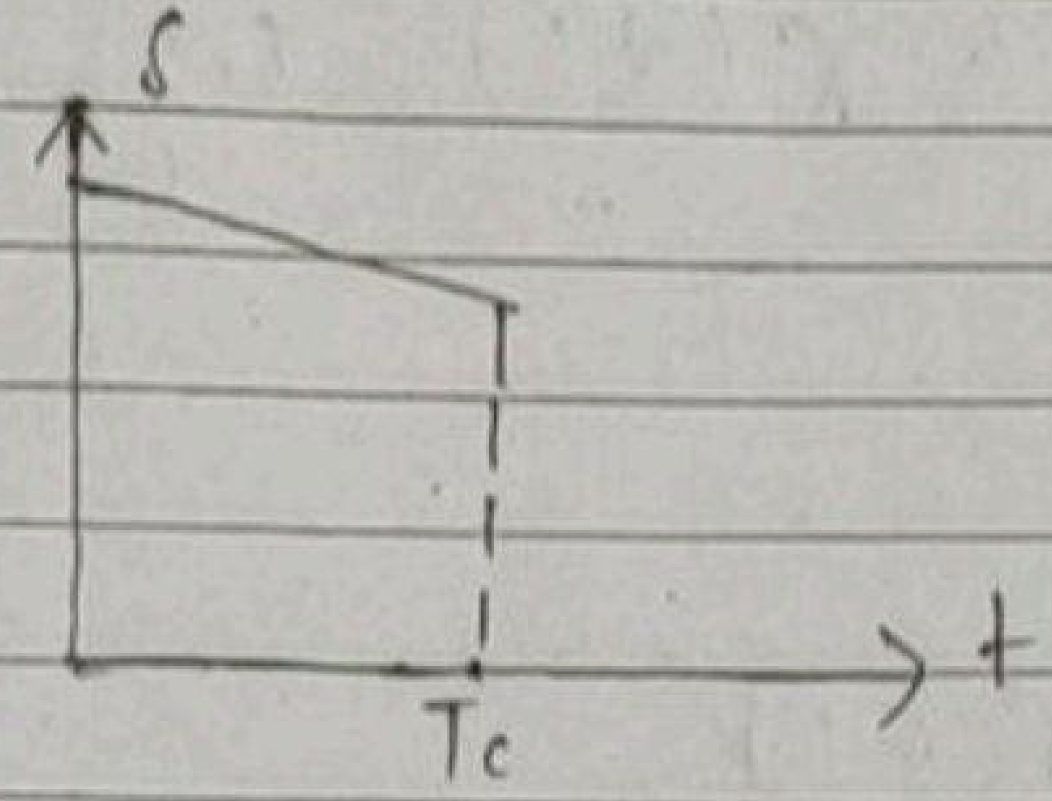


$F = S(4l) \times 2$

square frame square

Q) Soap Helps in cleaning clothes, because It decreases the surface tension of water

Surface tension $\propto \frac{1}{\text{water}}$



Surface Energy (E)

Energy per unit area $= \frac{E}{A} = S$

$E = S \times A$

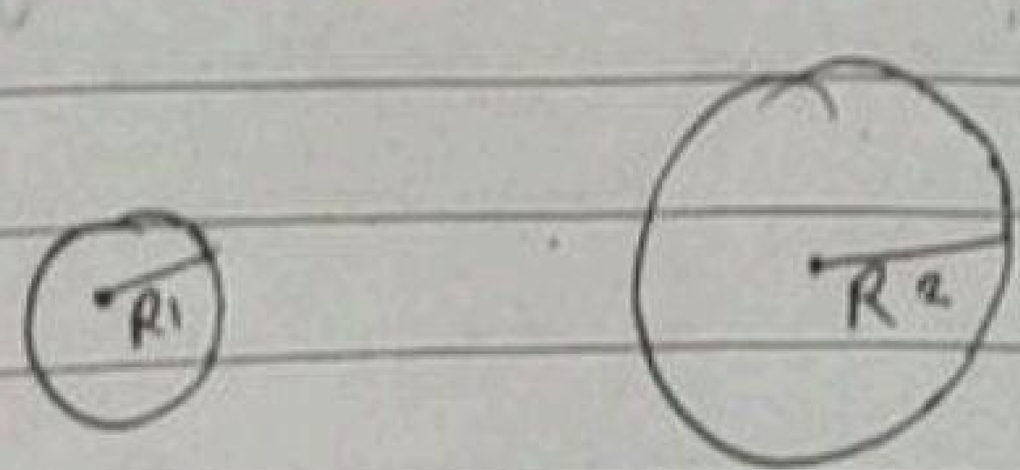
- Total energy $= E = S \times A$
- Energy per unit area $= S = \frac{E}{A}$

Total energy of a liquid drop

$E = S(4\pi R^2)$

Soap bubble

$E = S[4\pi R^2 \times 2] = 8S\pi R^2$



W done to Increase radius of liquid from R_1 to R_2 ΔE

$W = E_f - E_i$

$W = E_f - E_i \qquad S A_f - S A_i = S(A_f - A_i)$

Energy store to the molecules at the surface have extra energy as compared to the molecules in the interior is called surface energy.

Imp points:

→ The surface tensions arises due to intermolecular cohesive force.

→ It is surface tension due to which the surface of the liquid tends to contract and occupy surface area.

→ The surface tension acts along the tangent to the surface and is measured by normal force per unit length on an imaginary line drawn in the plane of the liquid surface, acting at right angles to this line.

$$S = \frac{F}{A} = \frac{E}{A}, \times 2$$

A rectangular film of liquid is extended from (4cm × 2cm) to (5cm × 4cm). If the work done is 3×10^{-4} J, the value of the surface tension of liquid.

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$A_i = 8cm^2 = 16 cm^2$
$A_f = 20 cm^2 = 40 cm^2$

$W = S(A_f - A_i)$
$3\times10^{-4} = S(40\times10^{-4} - 16\times10^{-4}]$
$3 = S[24]$
$S = \frac{1}{8} = 0.128 Nm^{-1}$

Q) The work done in blowing a soap Bubble of 10cm is tension of soap solⁿ is 0.03 N/m.

$W = S\times A$
$\frac{3}{100}\times[4\pi(\frac{1}{10})^2]\times2$
$\frac{3}{100}\times[4\times3.14\times\frac{1}{100}]\times2$
$37.6\times10^{-4}\times2 = 75.36\times10^{-4} J$

(7) ⊙ One liquid drop of Radius R - Break into n-small liquid drop - / spherical drop

$W = 4\pi SR^2[n^{1/3} - 1]$

⊙ Soap Buble

$W = 8\pi SR^2[n^{1/3} - 1]$

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Concept

$W = S[nA_f - A_i]$ $\Delta E = nE_f - E_i$

(2). if n small drop collease to make a large drop energy liberated / released

W = energy relased = $S[nA_i - A_f]$

(i) fraction of energy released

$\Delta E = E_1 - E_f$ divide E_1 both side

$$\frac{\Delta E}{E_1} = 1 - \frac{E_f}{E_i}$$

$$\left(100 \times \frac{\Delta E}{E_i}\right) = \left[1 - \frac{1}{n^{1/3}}\right] \times 100 = \frac{E_f - E_i}{E_i} \times 100$$

(i) $\frac{E_f}{E_i} = \frac{1}{n^{1/3}}$

Q) If T is the surface tension of a fluid then the energy needed to break a liquid drop of radius R into 64 equal drops is

$$W = 4\pi S R^2 [n^{1/3} - 1]$$
$$= 4\pi T R^2 [(64)^{1/3} - 1]$$
$$= 12\pi T R^2$$

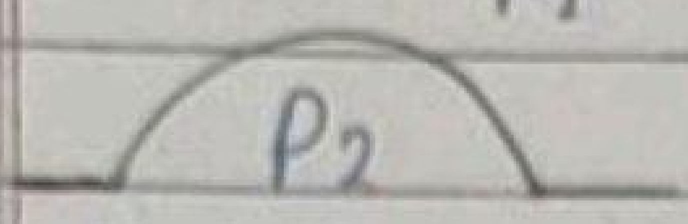
V_{Ti}

V_{T_f}

$R = n^{1/3}$

$V_{Tf} = V_i n^{1/3}$

Excess Pressure across a curved surface



$P_2 > P_1$

$\Delta P = \frac{2\,Tension}{R}$

Drop $\quad A = 4\pi R^2$

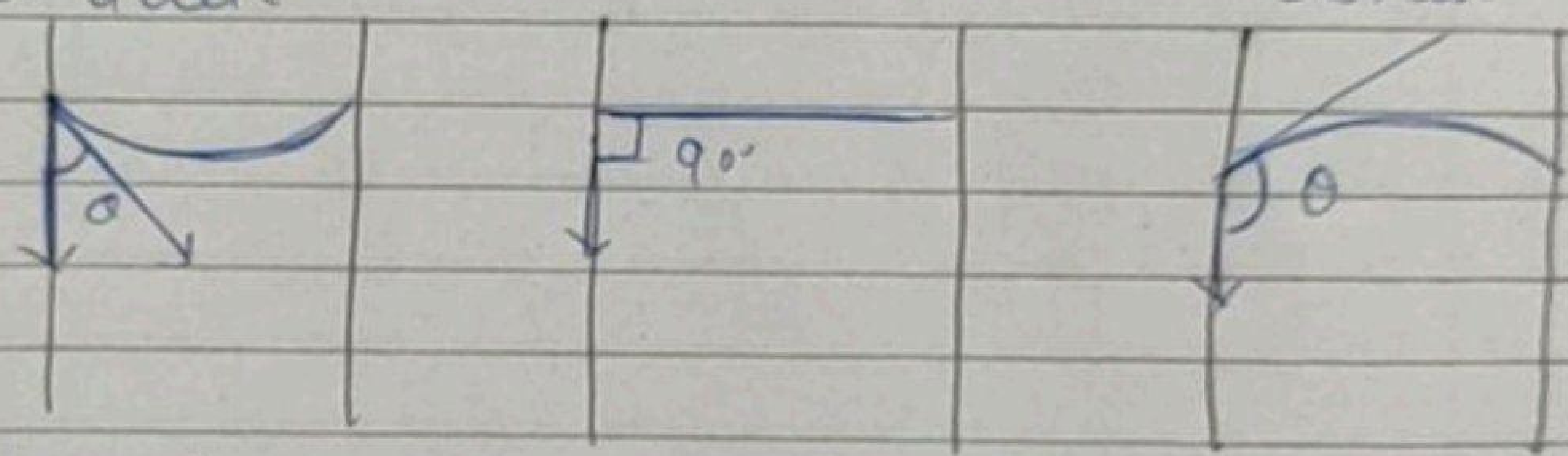
$\Delta P = \frac{2\,Tension}{R}$

Bubble

$\Delta P = \frac{4\,Tension}{R} \quad A = 8\pi R^2$

Angle of contact

θ - acute $\qquad$ Obtuse



Wetting $\qquad$ No wetting

Cohesive Force

Force of attraction between molecules of the same substance

Adhesive force

Force of attraction between molecules of the different substances.

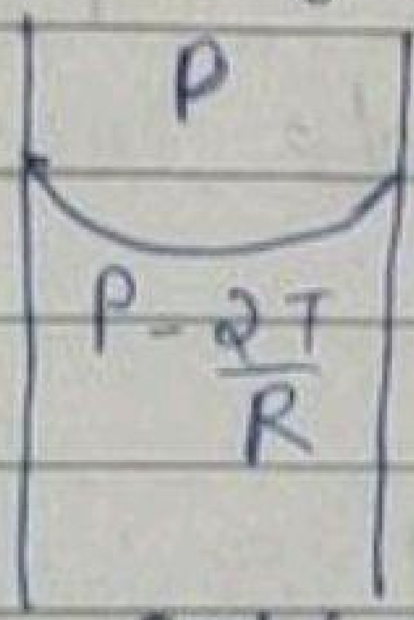
water

Adhesive > cohesive

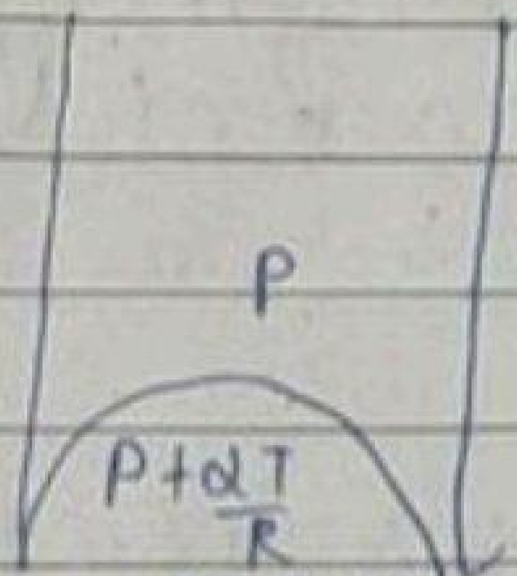
Hg

cohesive > Adhesive

Wettability of a surface by a liquid depends primarily on angle of contact between the surface and the liquid



Cappillary rise



capillary fall

Cappillary:

A very narrow glass tube that opens at both ends and with a fine bore hole is called a capillary tube

$$h\rho g = 2S/R = \frac{2S\cos\theta}{r}$$

S Surface Tension

θ angle of contact

r radius of capillary tube

R radius of menicus

ρ density of the liquid

In a capillary if radius is n times

Height 1/n times

Volume n times

mass n times

Capillary rise - Insufficient Height

$h \rho g = \frac{2 S \cos\theta}{r}$

Q) water rises to a Height h in capillary tube if the length of capillary tube above the surface of water is made less than h then.

water rises upto the top of capillary tube and stays there without overflow

If more than h Hota then, Overflowing like mountain

Capillary Application:

→ Swelling of wood in rainy season
→ Tissue paper soaks water